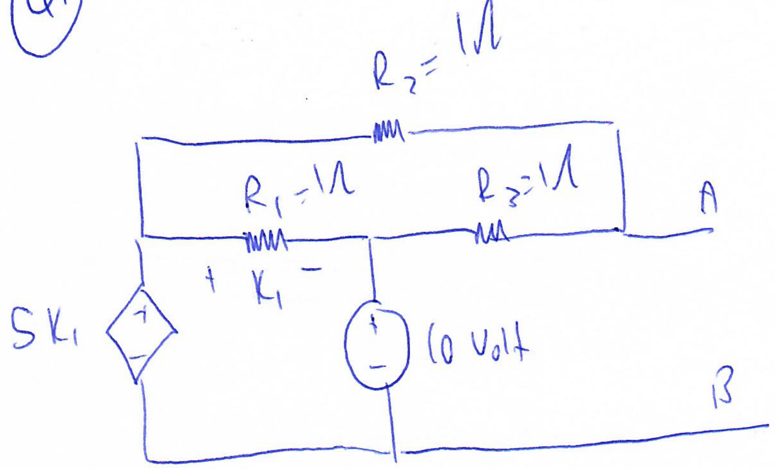
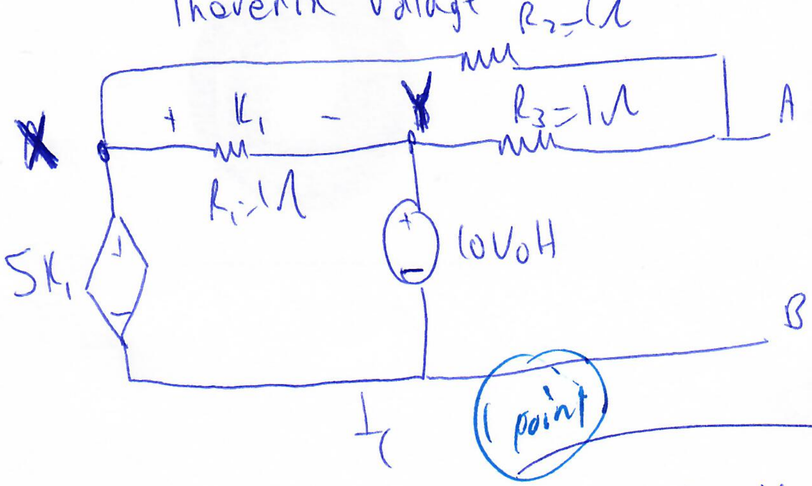
Q1

MECE 233 Midterm 2017-2018 fall



Thevenin Voltage



1 point

1 point

$Y = 10$ Volt $\quad X = 5K_1 \quad K_1 = X - Y \longrightarrow K_1 = 5K_1 - Y$

$Y = 4K_1$ 2 point

$\frac{X - A}{R_2} = \frac{A - Y}{R_3}$ 3 point $\quad X + Y = 2A$

$Y = 10 = 4K_1$

1 point $K_1 = 2.5$ Volt

$X = 12.5$ Volt

2 point

$X + Y = 2A \quad A = \frac{22.5}{2} = 11.25$

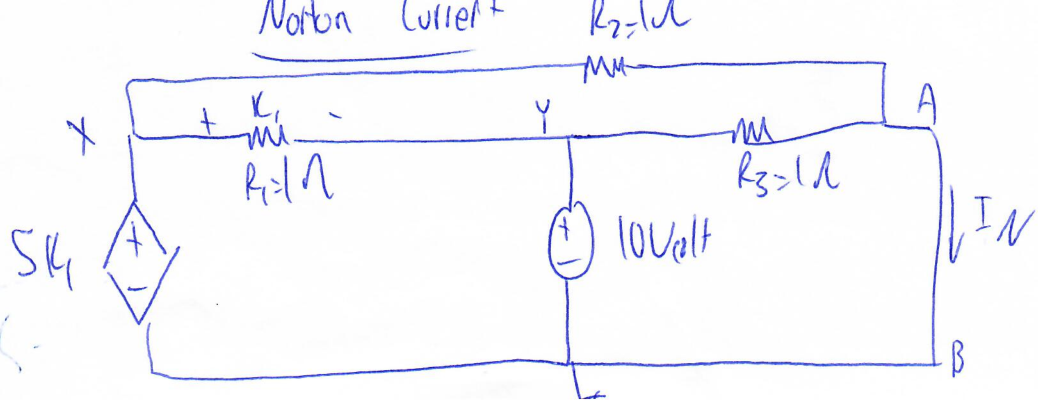
$12.5 + 10 = 2A$

$A = 11.25$ Volt

$A - B = V_{TH} = 11.25$ Volt

Norton Current

1 point



$Y = 10$ Volt $\quad A = B = 0$ Volt

3 point

$X = 5K_1 \quad X - Y = K_1 \quad 5K_1 - Y = K_1 \quad Y = 4K_1 = 10 \quad K_1 = 2.5$

$X = 12.5 \quad Y = 10$

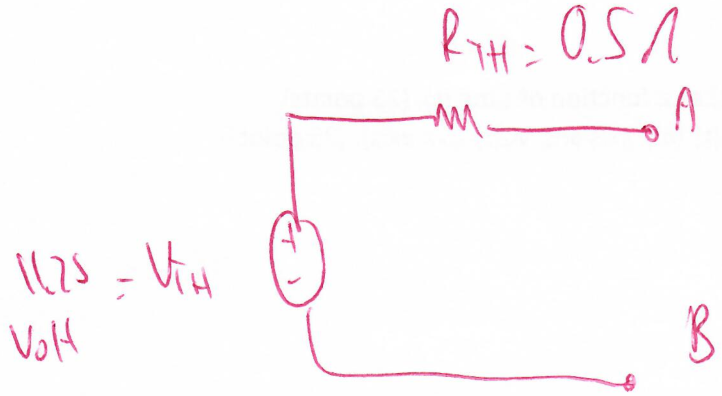
4 point $\quad$ 1 point

$$I_N = \frac{Y - A}{R_3} + \frac{X - A}{R_2} = \frac{10 - 0}{1} + \frac{12.5 - 0}{1} = 22.5 \text{ Amper}$$

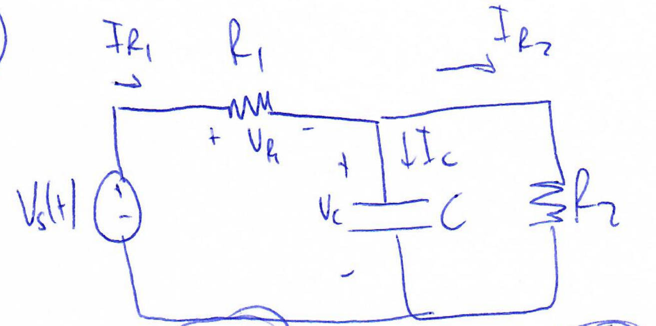
$V_{TH} = 11.25$ Volt $\qquad R_{TH} = \frac{11.25}{22.5} = 0.5\ \Omega$

$I_N = 22.5$ Amper

1 point



Q2



$V_s = V_{R_1} + V_c$ (2 point) $\qquad V_c = V_{R_2}$ (2 point)

$V_s = I_{R_1} R_1 + V_c$

$V_s = (I_c + I_{R_2}) R_1 + V_c$ (2 point) $\qquad V_s = \left[C\frac{dV_c}{dt} + \frac{V_c}{R_2} \right] R_1 + V_c$ (2 point) (2 point)

$V_s = C R_1 \frac{dV_c}{dt} + \frac{R_1}{R_2} V_c + V_c \qquad C R_1 \frac{dV_c}{dt} + \left[\frac{R_1 + R_2}{R_2}\right] V_c = V_s$

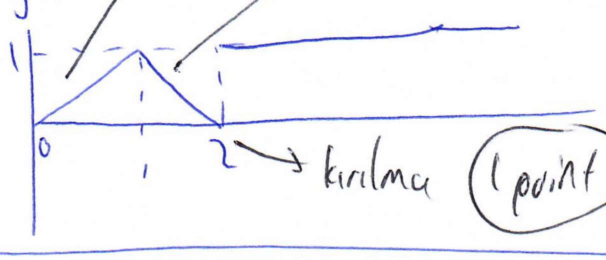
$$\boxed{\frac{dV_c}{dt} + \frac{(R_1 + R_2)}{C R_1 R_2} V_c = \frac{1}{C R_1} V_s}$$

5 point

Q3 $\quad y(t) = r(t) - 2r(t-1) + r(t-2) + u(t-2)$

2 point $\quad$ 3 point $\quad$ 1 point $\qquad$ 10 point

$y(t) =$



kırılma (1 point)

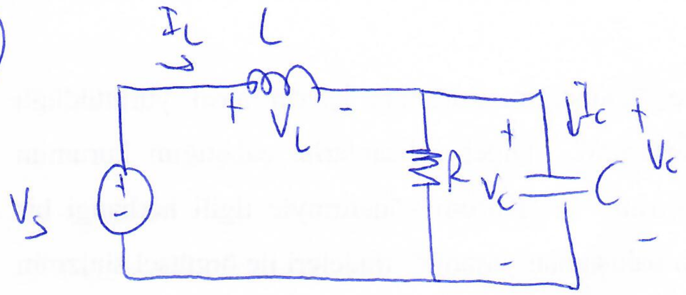
Q4 $\quad \delta(t) f(t) = \delta(t) f(0) = \delta(t) \cdot 1 = \delta(t)$

$f(t) = 5t^2 + 1$

$f(0) = 1$

3 point

Q5



1 point

(a) $V_s = V_L + V_c$ 2 point $\quad V_s = L\frac{dI_L}{dt} + V_c$ 2 point $\quad \boxed{\frac{dI_L}{dt} = \frac{V_s}{L} - \frac{V_c}{L}}$

$I_L = I_R + I_C$ 2 point

$I_L = \frac{V_c}{R} + C\frac{dV_c}{dt}$ 2 point $\quad \boxed{\frac{dV_c}{dt} = \frac{I_L}{C} - \frac{V_c}{CR}}$ 1 point

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{CR} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_s$$

(b) $\frac{d^2V_c}{dt^2} = \frac{1}{C}\frac{dI_L}{dt} - \frac{1}{CR}\frac{dV_c}{dt}$ 5

$\frac{d^2V_c}{dt^2} + \frac{1}{CR}\frac{dV_c}{dt} = \frac{1}{C}\left[\frac{V_s}{L} - \frac{V_c}{L}\right]$ 5

$$\boxed{\frac{d^2V_c}{dt^2} + \frac{1}{CR}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{1}{LC}V_s}$$

Q6

$$\frac{d^2V_c}{dt^2} + 7\frac{dV_c}{dt} + 10V_c = 10$$

Step ① $V_{cp} = K$ 3 point

$$\frac{d^2}{dt^2}K + 7\frac{d}{dt}K + 10K = 10$$ $K = 1$ $V_{cp} = 1$ Volt

1 point

Step ② char-equation $s^2 + 7s + 10 = 0 = (s+2)(s+5) = 0$ 1 point $s_1 = -2$ $s_2 = -5$ natural frequencies

$V_{ch} = K_1 e^{-2t} + K_2 e^{-5t}$ 1 point

Step ③

$V_c = V_{cp} + V_{ch} = 1 + K_1 e^{-2t} + K_2 e^{-5t}$ 1 point

Step ④

$V_c(0) = 0$ $1 + K_1 + K_2 = 0$ 1 point

$\frac{dV_c}{dt} = -2K_1 e^{-2t} - 5K_2 e^{-5t}$ $\frac{dV_c}{dt}(0) = -2K_1 - 5K_2 = 0$

$K_1 + K_2 = -1$ $2K_1 + 5K_2 = 0$ $K_1 = \frac{-5K_2}{2}$ 2 point

$-\frac{5}{2}K_2 + K_2 = -1$ $K_2 = \frac{2}{3}$ $K_1 = -\frac{5}{3}$

$-5K_2 + 2K_2 = -2$

$-3K_2 = -2$

$V_c(t) = 1 - \frac{5}{3}e^{-2t} + \frac{2}{3}e^{-5t}$ 2 point

Q6 $\cancel{\frac{d^2V_c}{dt^2} + A\frac{dV_c}{dt} + BV_c = 10}$ $s_{1,2} = \frac{-A \mp \sqrt{A^2-4B}}{2}$

(a) ~~Overdamped~~ $\cancel{A^2-4B>0}$

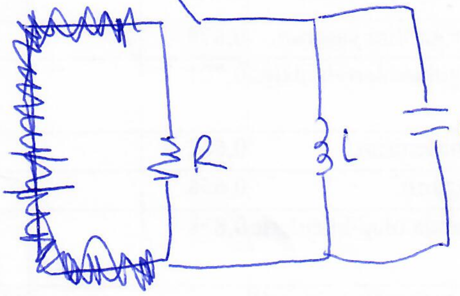
(b) ~~Critically-damped~~ $\cancel{A^2-4B=0 \quad A^2=4B}$

(c) ~~Underdamped~~ $\cancel{A^2-4B<0}$

(d) ~~Purely sinusoidal~~ $\cancel{A=0,\ B>0}$

Q7

$t=2\pi$



$V_c(0) = 1$ Volt

$I_L(0) = 0$ Ampere

$L = 1$ Henry

$C = 1$ Farad

$0 < t < 2\pi$

1 1

$I_L + I_C = 0$ $V_L = V_c = L\frac{dI_L}{dt}$

$1H = L$ $C = 1F$ V_c $\int I_L\, dt$

$I_L + C\frac{dV_c}{dt} = 0$ $I_L + CL\frac{d^2I_L}{dt^2} = 0$ 2 $\rightarrow s^2+1=0$ 1 $s_{1,2} = \mp j$

(a)

$\frac{d^2I_L}{dt^2} + \frac{1}{LC}I_L = 0$ $I_L(t) = K_1\sin(t) + K_2\cos(t)$

$\frac{d^2I_L}{dt^2} + I_L = 0 \longrightarrow$

$L\frac{dI_L}{dt} = V_c$ $\frac{dI_L}{dt} = \frac{V_c}{L}$ $\frac{dI_L}{dt}(0) = \frac{V_c(0)}{L} = \frac{1}{1} = 1$ 1

$I_L(0) = 0$ Ampere

$I_L(0) = 0 = K_1\sin(0) + K_2\cos(0)$

$0 = K_2\cos(0) \rightarrow K_2 = 0$ $\Rightarrow I_L = K_1\sin(t)$

$\frac{dI_L}{dt}(0) = K_1\cos(0) = 1$ $K_1 = 1$ $I_L(t) = \sin(t)$ 2

$I_L(2\pi^-) = \sin(2\pi) = 0$ Ampere

$V_C = L\frac{dI_L}{dt} = 1 \frac{d}{dt}\sin(t) = \cos(t)$

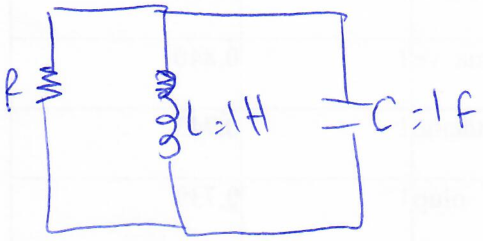
$V_C(2\pi^-) = \cos(2\pi^-) = 1$ Volt

1

$V_C(t) = L\frac{dI_L}{dt} = 1\frac{d}{dt}\sin(t) = \cos(t)$

1

$t > 2\pi$



$$\frac{V_C}{R} + C\frac{dV_C}{dt} + I_L = 0$$

$$\frac{1}{R}\frac{dV_C}{dt} + C\frac{d^2V_C}{dt^2} + \frac{dI_L}{dt} = 0$$

$R = \frac{1}{2}\Omega$

3

$$\frac{1}{R}\frac{dV_C}{dt} + C\frac{d^2V_C}{dt^2} + \frac{V_C}{L} = 0$$

$L = 1\,H$

$C = 1\,F$

$$\frac{d^2V_C}{dt^2} + \frac{1}{RC}\frac{dV_C}{dt} + \frac{1}{LC}V_C = 0$$

homogeneous equations

$$\frac{d^2V_C}{dt^2} + 2\frac{dV_C}{dt} + V_C = 0 \qquad \text{similarly} \qquad \frac{d^2I_L}{dt^2} + 2\frac{dI_L}{dt} + I_L = 0$$

1

char-equation $= (s^2 + 2s + 1) = (s+1)^2 \to s_1 = -1,\ s_2 = -1 \to$ critically damped

$\cancel{I_L(t) = M_1 + M_2 e^{-t} t}$

$\cancel{I_L(t) = M_1 e^{-t} + M_2 t e^{-t}}$

$\cancel{I_L(2\pi) = \sin(2\pi) = 0}$

$\cancel{I_L(2\pi) = M_1 e^{-2\pi} + M_2 2\pi e^{-2\pi}}$

1

$$I_L(t) = M_1 e^{-(t-2\pi)} + M_2 (t-2\pi)\, e^{-(t-2\pi)}$$

$I_L(2\pi^+) = I_L(2\pi^-) = 0$ Ampere

1

$$L\frac{dI_L}{dt} = V_C \qquad \left.\frac{dI_L}{dt}\right|_{t=2\pi^+} = \frac{V_C(2\pi^+)}{L} = \frac{1}{1}$$

$0 = I_L(2\pi^+) = M_1 = 0$ 1

then

$I_L(t) = M_2(t-2\pi)e^{-(t-2\pi)} = M_2 e^{2\pi}(t-2\pi)e^{-t}$

$\cancel{\frac{dI_L}{dt} = M_2 e^{-(t-2\pi)} + (-1)(t-2\pi)e^{-(t-2\pi)} M_2}$

$\frac{dI_L}{dt} = M_2 e^{2\pi}\left[1e^{-t} + (t-2\pi)(-1)e^{-t}\right]$

$\left.\frac{dI_L}{dt}\right|_{t=2\pi} = M_2 = 1$ 1

$I_L(t) = (t-2\pi)e^{-(t-2\pi)} = (t-2\pi)e^{2\pi}e^{-t}$

$L\frac{dI_L}{dt} = V_C = \cancel{e^{-(t-2\pi)} + (t-2\pi)(-1)e^{-(t-2\pi)}}$

$= e^{2\pi}\left[1e^{-t} + (-1)e^{-t}(t-2\pi)\right]$ 2

$V_C(t) = e^{-(t-2\pi)} - (t-2\pi)e^{-(t-2\pi)}$

$$V_C(t) = \begin{cases} \cos(t), & 0 \le t < 2\pi \\ e^{-(t-2\pi)} - (t-2\pi)e^{-(t-2\pi)}, & 2\pi \le t \end{cases}$$

$$I_L(t) = \begin{cases} \sin(t), & 0 \le t < 2\pi \\ (t-2\pi)e^{-(t-2\pi)}, & 2\pi \le t \end{cases}$$