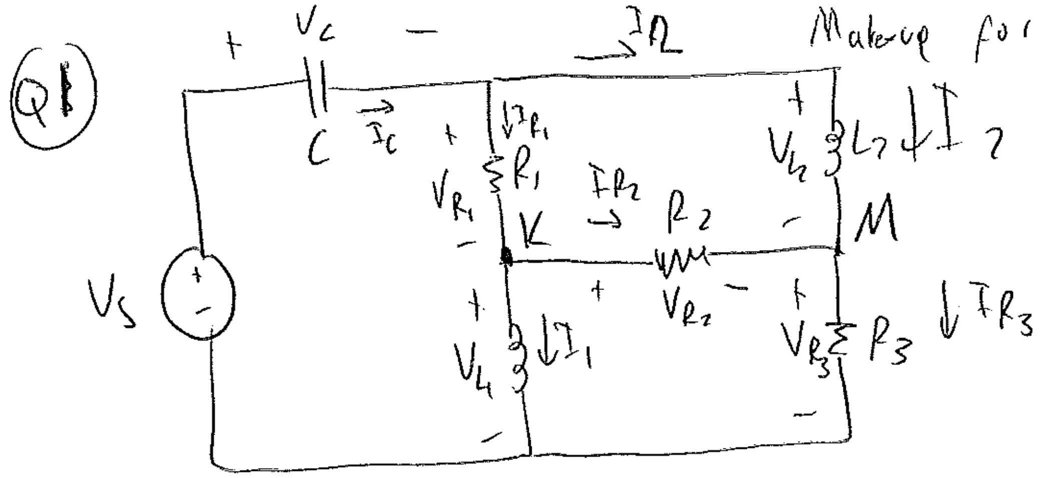
Q1 Makeup for midterm 2017-2018 fall



$R_1 = R_2 = R_3 = 1\Omega$

$L_1 = L_2 = 1H$

$C = 2F$

$$V_s = V_c + V_{R_1} + V_{L_1}$$

$$V_s = V_c + I_{R_1} R_1 + L_1 \frac{dI_1}{dt}$$

$$I_{R_1} = I_c - I_2$$

$$V_s = V_c + (I_c - I_2) R_1 + L_1 \frac{dI_1}{dt}$$

$$V_s = V_c + \left(C\frac{dV_c}{dt} - I_2\right) R_1 + L_1 \frac{dI_1}{dt}$$

$$V_s = V_c + \left[2\frac{dV_c}{dt} - I_2\right] \times 1 + \frac{dI_1}{dt}$$

$$V_s = V_c + V_{L_2} + V_{R_3}$$

$$V_s = V_c + L_2 \frac{dI_2}{dt} + I_{R_3} R_3$$

$$I_{R_3} = I_c - I_1$$

$$V_s = V_c + L_2 \frac{dI_2}{dt} + R_3 (I_c - I_1)$$

$$V_s = V_c + L_2 \frac{dI_2}{dt} + R_3 \left[C\frac{dV_c}{dt} - I_1\right]$$

$$V_s = V_c + \frac{dI_2}{dt} + 1 \times \left[2\frac{dV_c}{dt} - I_1\right]$$

$$I_c = I_{R_1} + I_2 = I_1 + I_{R_2} + I_2$$

$$I_c = I_1 + I_2 + I_{R_2}$$

$$I_{R_2} = \frac{K - M}{R_2} = \frac{V_{L_1} - M}{R_2}$$

$$M = V_s - V_c - V_{L_2}$$

$$C\frac{dV_c}{dt} = I_1 + I_2 + \frac{V_{L_1} - (V_s - V_c + V_{L_2})}{R_2}$$

$$\boxed{2\frac{dV_c}{dt} = I_1 + I_2 + \frac{L_1\frac{dI_1}{dt} + L_2\frac{dI_2}{dt} + V_c - V_s}{1}}$$

$$2\frac{dV_c}{dt} = I_1 + I_2 + \frac{dI_1}{dt} + \frac{dI_2}{dt} + V_c - V_s$$

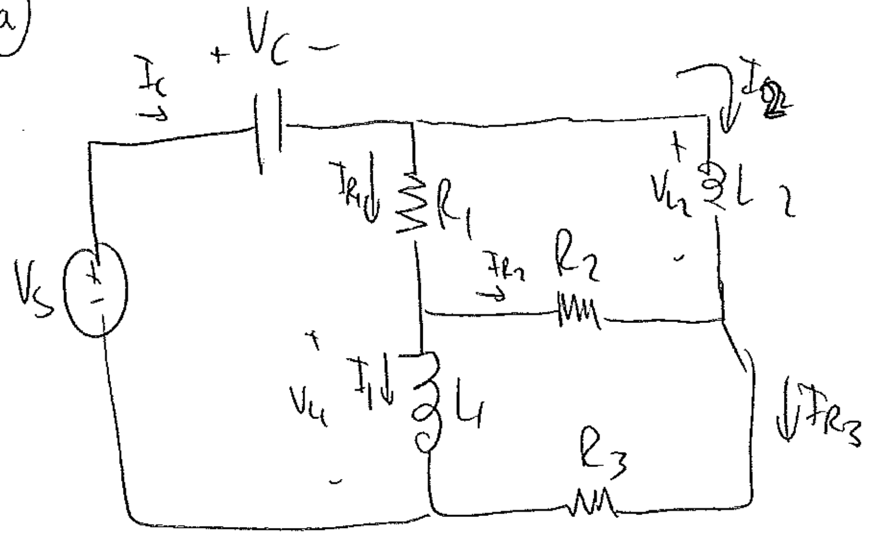
$C = 2$ farad

$L_1 = 1H$, $L_2 = 1H$

$R_1 = 1\Omega$ $R_2 = 1\Omega$ $R_3 = 1\Omega$

$C = 2$ farad

(a)



$$I_{R_1} = \frac{V_s - V_C - V_{L_1}}{R_1} = \frac{V_s - V_C - L_1\frac{dI_1}{dt}}{R_1} = \frac{V_s - V_C - \frac{dI_1}{dt}}{1} = \boxed{V_s - V_C - \frac{dI_1}{dt} = I_{R_1}}$$ 1 puan

$I_C = I_{R_1} + I_2$ 1 puan

$$C\frac{dV_C}{dt} = V_s - V_C - \frac{dI_1}{dt} + I_2$$

$$\boxed{2\frac{dV_C}{dt} = V_s - V_C - \frac{dI_1}{dt} + I_2}$$

* 2 puan

$$\frac{V_{L_1} - (V_s - V_C - V_{L_2})}{R_2} = I_{R_2}$$ 1 puan

$$\frac{L_1\frac{dI_1}{dt} - (V_s - V_C - L_2\frac{dI_2}{dt})}{R_2} = I_{R_2}$$

$$\boxed{\frac{dI_1}{dt} + \frac{dI_2}{dt} - V_s + V_C = I_{R_2}}$$ 2 puan

$I_{R_1} = I_1 + I_{R_2}$ 1 puan

$$V_s - V_C - \frac{dI_1}{dt} = I_1 + \frac{dI_1}{dt} + \frac{dI_2}{dt} - V_s + V_C$$

$$\boxed{2V_s = 2\frac{dI_1}{dt} + I_1 + \frac{dI_2}{dt} + 2V_C}$$ ** 2 puan

$$I_{R_3} = \frac{V_s - V_C - V_{L_2}}{R_3} = \frac{V_s - V_C - L_2\frac{dI_2}{dt}}{R_3}$$

$$I_{R_3} = \frac{V_s - V_C - \frac{dI_2}{dt}}{1}$$ 1 puan

$I_1 + I_{R_3} = I_C$ 1 puan

$$I_1 + V_s - V_C - \frac{dI_2}{dt} = C\frac{dV_C}{dt}$$

$$\boxed{2\frac{dV_c}{dt} = V_s - V_c - \frac{dI_2}{dt} + I_1}$$

2 puan

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* and ***

$$V_s - V_c - \frac{dI_2}{dt} + I_1 = V_s - V_c - \frac{dI_1}{dt} + I_2 \Rightarrow \frac{dI_1}{dt} + I_1 = \frac{dI_2}{dt} + I_2$$

$$\frac{dI_1}{dt} = \frac{dI_2}{dt} + I_2 - I_1 \quad \text{and} \quad \frac{dI_2}{dt} = \frac{dI_1}{dt} + I_1 - I_2$$

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$$2V_s = 2\left[\frac{dI_2}{dt} + I_2 - I_1\right] + I_1 + \frac{dI_2}{dt} + 2V_c \qquad 2V_s = 2\frac{dI_1}{dt} + I_1 + \left(\frac{dI_1}{dt} + I_1 - I_2\right) + 2V_c$$

$$2V_s = 3\frac{dI_2}{dt} + 2I_2 - I_1 + 2V_c \qquad 2V_s = 3\frac{dI_1}{dt} + 2I_1 - I_2 + 2V_c$$

2 puan 2 puan

$$\boxed{\frac{dI_2}{dt} = \frac{2}{3}V_s - \frac{2}{3}I_2 + \frac{1}{3}I_1 - \frac{2}{3}V_c} \qquad \boxed{\frac{dI_1}{dt} = \frac{2}{3}V_s - \frac{2}{3}I_1 + \frac{1}{3}I_2 - \frac{2}{3}V_c}$$

$$2\frac{dV_c}{dt} = V_s - V_c - \frac{dI_1}{dt} + I_2 = V_s - V_c - \left[\frac{2}{3}V_s - \frac{2}{3}I_1 + \frac{1}{3}I_2 - \frac{2}{3}V_c\right] + I_2$$

2 puan

$$\boxed{2\frac{dV_c}{dt} = \frac{1}{3}V_s - \frac{1}{3}V_c + \frac{2}{3}I_1 + \frac{2}{3}I_2} \qquad \frac{dV_c}{dt} = \frac{1}{6}V_s - \frac{1}{6}V_c + \frac{1}{3}I_1 + \frac{1}{3}I_2$$

2 puan

$$\begin{bmatrix} \frac{dI_1}{dt} \\ \frac{dI_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{6} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_c \end{bmatrix} + \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{1}{6} \end{pmatrix} V_s$$

(b)

$$\frac{dV_c}{dt} = \frac{1}{3} I_1 + \frac{1}{3} I_2 - \frac{1}{6} V_c + \frac{1}{6} V_s$$

$$\frac{d^2V_c}{dt^2} = \frac{1}{3}\frac{dI_1}{dt} + \frac{1}{3}\frac{dI_2}{dt} - \frac{1}{6}\frac{dV_c}{dt} + \frac{1}{6}\frac{dV_s}{dt}$$

$$\frac{d^3V_c}{dt^3} = \frac{1}{3}\frac{d^2I_1}{dt^2} + \frac{1}{3}\frac{d^2I_2}{dt^2} - \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} = \frac{1}{3}\left[\frac{d^2I_1}{dt^2} + \frac{d^2I_2}{dt^2}\right] + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} = \frac{1}{3}\left[-\frac{1}{3}\frac{dI_1}{dt} - \frac{1}{3}\frac{dI_2}{dt} - \frac{4}{3}\frac{dV_c}{dt} + \frac{4}{3}\frac{dV_s}{dt}\right] + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{4}{9}\frac{dV_c}{dt} = -\frac{1}{9}\left[\frac{dI_1}{dt} + \frac{dI_2}{dt}\right] + \frac{4}{9}\frac{dV_s}{dt} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^2I_1}{dt^2} = -\frac{2}{3}\frac{dI_1}{dt} + \frac{1}{3}\frac{dI_2}{dt} - \frac{2}{3}\frac{dV_c}{dt} + \frac{2}{3}\frac{dV_s}{dt}$$

$$\frac{d^2I_2}{dt^2} = \frac{1}{3}\frac{dI_1}{dt} - \frac{2}{3}\frac{dI_2}{dt} - \frac{2}{3}\frac{dV_c}{dt} + \frac{2}{3}\frac{dV_s}{dt}$$

$$\frac{d^2I_1}{dt^2} + \frac{d^2I_2}{dt^2} = -\frac{1}{3}\frac{dI_1}{dt} - \frac{1}{3}\frac{dI_2}{dt} - \frac{4}{3}\frac{dV_c}{dt} + \frac{4}{3}\frac{dV_s}{dt}$$

$$\underset{1}{L_1}\frac{dI_1}{dt} + \underset{1}{L_2}\frac{dI_2}{dt} = V_{L_1} + V_{L_2} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$V_s - V_c = V_{L_2} - V_{R_2} + V_{L_1}$$

$$V_s - V_c + V_{R_2} = V_{L_1} + V_{L_2} = L_1\frac{dI_1}{dt} + L_2\frac{dI_2}{dt}$$

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$$V_s - V_c + R_2[I_c - I_2 - I_1] = V_{L_1} + V_{L_2} = \frac{dI_1}{dt} + \frac{dI_c}{dt}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{4}{9}\frac{dV_c}{dt} = -\frac{1}{9}\left[V_s - V_c + 1\left[C\frac{dV_c}{dt} - I_2 - I_1\right]\right] + \frac{4}{9}\frac{dV_s}{dt} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{4}{9}\frac{dV_c}{dt} + \frac{1}{9}V_C = -\frac{1}{9}V_s - \frac{2}{9}\frac{dV_c}{dt} + \frac{1}{9}[I_1+I_2] + \frac{4}{9}\frac{dV_s}{dt} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{6}{9}\frac{dV_c}{dt} + \frac{1}{9}V_C = -\frac{1}{9}V_s + \frac{1}{9}(I_1+I_2) + \frac{4}{9}\frac{dV_s}{dt} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{dV_c}{dt} = \frac{1}{3}[I_1+I_2] - \frac{1}{6}V_c + \frac{1}{6}V_s$$

$$\boxed{I_1+I_2 = 3\frac{dV_c}{dt} + \frac{1}{2}V_C - \frac{1}{2}V_s}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{6}{9}\frac{dV_c}{dt} + \frac{1}{9}V_C = -\frac{1}{9}V_s + \frac{1}{9}\left[3\frac{dV_c}{dt} + \frac{1}{2}V_C - \frac{1}{2}V_s\right] + \frac{4}{9}\frac{dV_s}{dt} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

$$\frac{d^3V_c}{dt^3} + \frac{1}{6}\frac{d^2V_c}{dt^2} + \frac{1}{3}\frac{dV_c}{dt} + \frac{1}{18}V_C = -\frac{1}{6}V_s + \frac{4}{9}\frac{dV_s}{dt} + \frac{1}{6}\frac{d^2V_s}{dt^2}$$

(Q2) $\ddot{x} + x = \sin(t)$

char equation $s^2+1=0$, natural frequencies $s_{1,2} = \mp j$

(a) $x_n = K_1 \sin(t) + K_2 \cos(t)$

(b) $x_p = t\left(M_1 \cos(t) + M_2 \sin(t)\right)$ (the order is increased w.r.t x_n)

$\ddot{x}_p + x_p = \sin(t)$

$\dot{x}_p = \left[M_1 \cos(t) + M_2 \sin(t)\right] + t\left[-M_1 \sin(t) + M_2 \cos(t)\right]$

$\ddot{x}_p = \left[-M_1 \sin(t) + M_2 \cos(t)\right] + \left[-M_1 \sin(t) + M_2 \cos(t)\right]$
$\quad + t\left(-M_1 \cos(t) - M_2 \sin(t)\right)$

$\ddot{x}_p + x_p = \left[-2M_1 \sin(t) + 2M_2 \cos(t)\right] + t\left(-M_1\cos(t) - M_2 \sin(t)\right)$
$\quad + t\left[M_1 \cos(t) + M_2 \sin(t)\right] = \sin(t)$

$-2M_1 \sin(t) + 2M_2 \cos(t) = \sin(t)$ $\qquad M_2 = 0$

$M_1 = -\frac{1}{2}$

$x_p = -\frac{1}{2} t \cos(t)$ //

(c) $x = x_p + x_n = K_1 \sin(t) + K_2 \cos(t) - \frac{1}{2} t \cos(t)$

(d) $x(0) = 5 = K_1 \sin(0) + K_2 \cos(0) - \frac{1}{2} \times 0 \times \cos(0)$

$K_2 = 5.5$

$\dot{x}(0) = 2 \Rightarrow +K_1 \cos(t) - K_2 \sin(t) - \frac{1}{2}\left[\cos(t) - t\sin(t)\right] = \dot{x}(t)$

$\dot{x}(0) = 2 \quad \dot{x}(0) = K_1 \cos(0) - K_2 \sin(0) - \frac{1}{2}\left[\cos(0) - 0\sin(0)\right]$

$2 = K_1 - \frac{1}{2} \qquad K_1 = \frac{5}{2}$ //

result
$x(t) = \frac{5}{2}\sin(t) + \frac{11}{2}\cos(t) - \frac{1}{2} t\cos(t)$ //