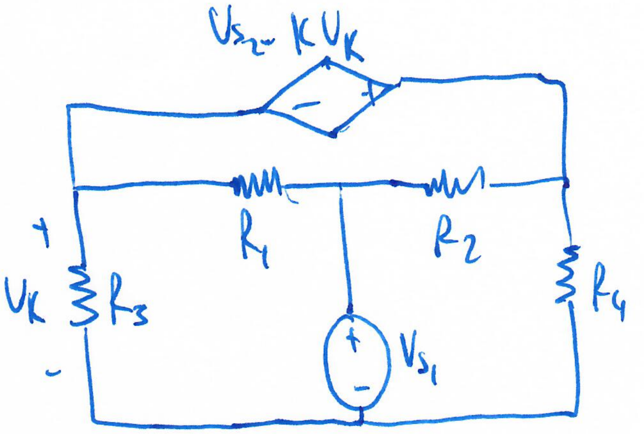
MECE 233 Final

2017-2018 fall

Q1



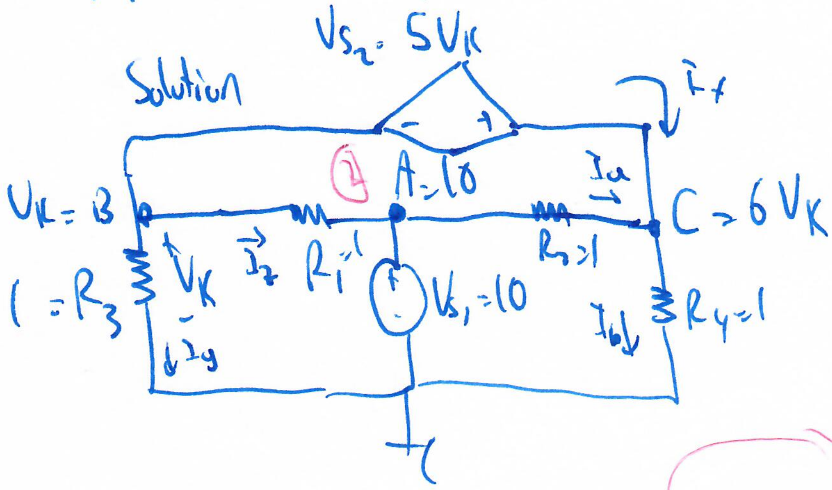
$R_1 = R_2 = R_3 = R_4 = 1\,\Omega$

$K = 5$

$V_{s_1} = 10$ Volt

~~what is the power dissipated over R_4~~ ; what is the power of V_{s_2}

Solution



3 puan

$I_x = -(I_y + I_z) = I_b - I_a$ 3 puan

$-\left(\frac{V_K}{1} + \frac{V_K - A}{1}\right) = \frac{6V_K}{1} - \frac{A - 6V_K}{1}$ 3 puan

$-V_K - V_K + A = 12V_K - A$

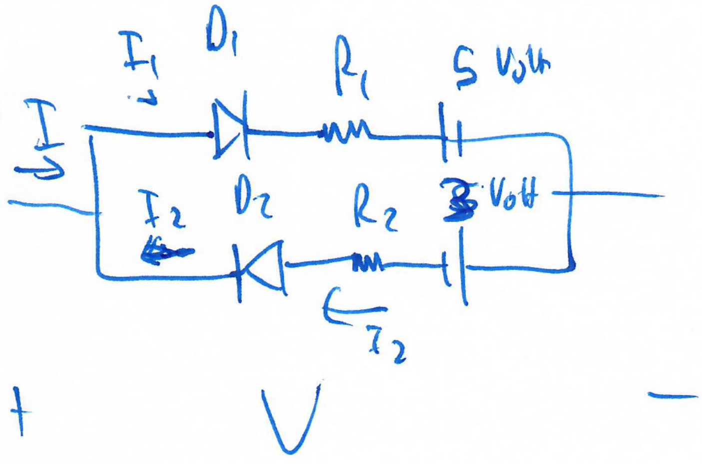
$2A = 14V_K$

$20 = 14V_K \quad V_K = \frac{20}{14} = \frac{10}{7}$ Volt 2 puan

$I_x = I_b - I_a = \frac{6V_K}{1} - \frac{10 - 6V_K}{1} = 6 \times \frac{10}{7} - \frac{10 - 6 \times \frac{10}{7}}{1} = \frac{12 \times 10}{7} - 10 = \frac{50}{7}$ Ampere 1 puan

$V_{s_2} = 5V_K = 5 \times \frac{10}{7} = \frac{50}{7} \quad P_{V_{s_2}} = V_{s_2} \times I_x = \frac{2500}{49}$ Watt 1 puan

Q2



$R_1 = 1\Omega$

$R_2 = 1\Omega$

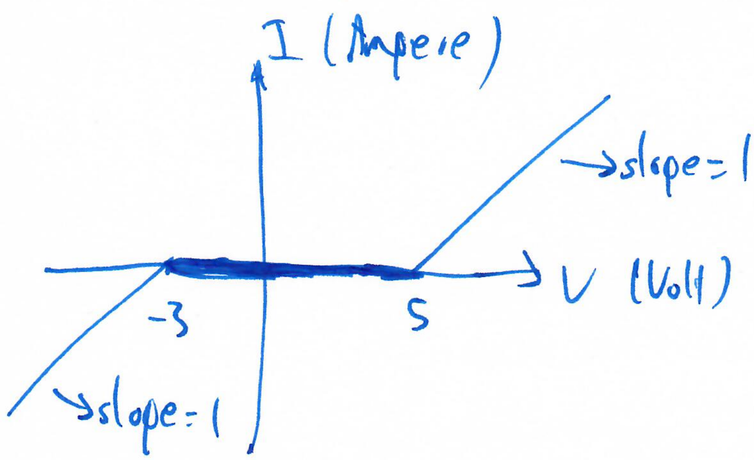
$I = I_1 - I_2$

$I_1 = \frac{V-5}{R_1}$ $\quad I_2 = \frac{-3-V}{R_2}$

if $V > 5$ $\quad I = I_1 = \frac{V-5}{1}$ 3

if $-3 < V < 5$ $\quad I = 0$ 2

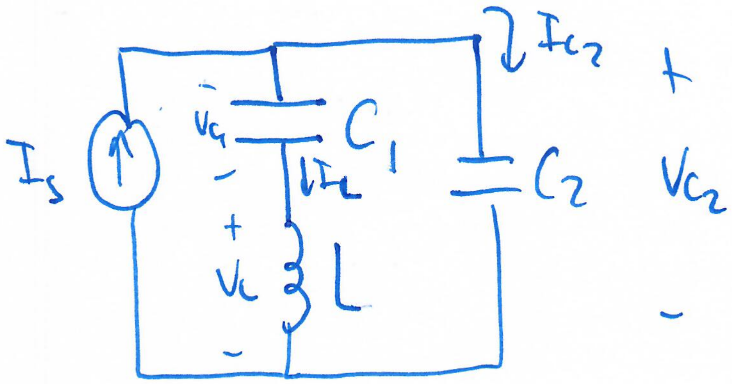
if $V < -3$ $\quad I = -I_2 = \frac{V+3}{1}$ 3



5 point

Q2 Q3

(a)



1 puan

$I_s = I_L + I_{c_2}$

$I_L = I_{c_1} = C_1 \frac{dV_{c_1}}{dt}$

1 puan

$$\boxed{\frac{dV_{c_1}}{dt} = \frac{I_L}{C_1}}$$

$I_s = I_L + C_2 \frac{dV_{c_2}}{dt}$

1 puan

$$\boxed{\frac{dV_{c_2}}{dt} = \frac{I_s}{C_2} - \frac{I_L}{C_2}}$$

$V_{c_1} + V_L = V_{c_2}$

1 puan

$V_{c_2} = L \frac{dI_L}{dt} + V_{c_1}$

1 puan

$$\boxed{\frac{dI_L}{dt} = \frac{V_{c_2}}{L} - \frac{V_{c_1}}{L}}$$

3 puan

$$\begin{bmatrix} \frac{dV_{c_1}}{dt} \\ \frac{dV_{c_2}}{dt} \\ \frac{dI_L}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & \frac{1}{C_1} \\ 0 & 0 & -\frac{1}{C_2} \\ -\frac{1}{L} & +\frac{1}{L} & 0 \end{bmatrix}}_{A} \begin{bmatrix} V_{c_1} \\ V_{c_2} \\ I_L \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{C_2} \\ 0 \end{bmatrix}}_{B} I_s$$

(b) $C_1 = C_2 = 1F$

$L = 2$ Henry

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -\frac{1}{2} & +\frac{1}{2} & 0 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & 0 & -1 \\ 0 & s & 1 \\ +\frac{1}{2} & -\frac{1}{2} & s \end{bmatrix}$$

2 puan

$$\det(sI-A) = (-1)^{1+1} s \begin{vmatrix} s & 1 \\ -\frac{1}{2} & s \end{vmatrix} + (-1)^{1+2} 0 \begin{vmatrix} 0 & 1 \\ -\frac{1}{2} & s \end{vmatrix} + (-1)^{1+3} (-1) \begin{vmatrix} 0 & s \\ +\frac{1}{2} & -\frac{1}{2} \end{vmatrix}$$

4 puan

$$\det(sI-A) = s\left(s^2 + \frac{1}{2}\right) + 0 + (-1)\left[0 - \left(+\frac{1}{2}\right)s\right]$$

$$= s\left(s^2 - \frac{1}{2}\right) + \left(+\frac{s}{2}\right) = s\left(s^2 + \frac{1}{2}\right) + \frac{s}{2}$$

$$= s^3 + \frac{s}{2} + \frac{s}{2} = s^3 + s = s(s^2+1)$$

$s_1 = 0 \quad s_{2,3} = \mp j$

2 puan

birdişat 10

(c) $\frac{dV_{c_1}}{dt} = I_L \qquad \frac{dV_{c_2}}{dt} = I_s - I_L \qquad \frac{dI_L}{dt} = \frac{V_{c_2}}{2} - \frac{V_{c_1}}{2}$

$$\frac{d^2V_{c_1}}{dt^2} = \frac{dI_L}{dt} \rightarrow \frac{d^3V_c}{dt^3} = \frac{d^2I_L}{dt^2} = \frac{d}{dt}\left[\frac{dI_L}{dt}\right] = \frac{d}{dt}\left[\frac{V_{c_2}}{2} - \frac{V_{c_1}}{2}\right]$$

$$\frac{d^3V_{c_1}}{dt^3} = \frac{1}{2}\frac{dV_{c_2}}{dt} - \frac{1}{2}\frac{dV_{c_1}}{dt} \rightarrow \frac{d^3V_{c_1}}{dt^3} + \frac{1}{2}\frac{dV_{c_1}}{dt} = +\frac{1}{2}\left[I_s - I_L\right]$$

$$\frac{d^3V_{c_1}}{dt^3} + \frac{1}{2}\frac{dV_{c_1}}{dt} = +\frac{1}{2}I_s - \frac{1}{2}I_L \qquad \frac{d^3V_{c_1}}{dt^3} + \frac{1}{2}\frac{dV_{c_1}}{dt} = +\frac{1}{2}I_s - \frac{1}{2}\frac{dV_{c_1}}{dt}$$

$$\boxed{\frac{d^3V_{C_1}}{dt^3} + \frac{dV_{C_1}}{dt} = \frac{1}{2} I_s}$$ $\rightarrow$ 4 puan

04 05 $\dddot{x} + \dot{x} = 5t$ $\quad x(0) = 1 \quad \dot{x}(0) = 1 \quad \ddot{x}(0) = 1$

a) char equation $s^3 + s = 0$ 2

b) natural frequencies $s(s^2+1) = 0 \quad s_1 = 0 \quad s_{2,3} = \mp j$ 2

c) $x_h = K_1 + K_2 \cos(t) + K_3 \sin(t)$ 2

d) $x_p = Mt + N$ 2

$\frac{d^3}{dt^3} x_p + \frac{d}{dt} x_p = 5t$ $\xrightarrow{\text{hence}}$ $\frac{d^3}{dt^3} Mt^2 + \frac{d}{dt} Mt^2 = 5t$

$0 + M = 5t$ $\quad x_p = Mt^2 \quad$ $0 + 2Mt = 5t \quad M = \frac{5}{2}$

not posible

e) $x = K_1 + K_2 \cos(t) + K_3 \sin(t) + \frac{5}{2} t^2$ 8 puan

$x(0) = K_1 + K_2 = 1$

$\dot{x} = -K_2 \sin(t) + K_3 \cos(t) + 5t$

$\dot{x}(0) = K_3 = 1$

$\ddot{x} = -K_2 \cos(t) - K_3 \sin(t) + 5$

$\ddot{x}(0) = -K_2 + 5 = 1 \quad K_2 = 4$

$x(t) = -3 + 4\cos(t) + \sin(t) + \frac{5}{2} t^2$

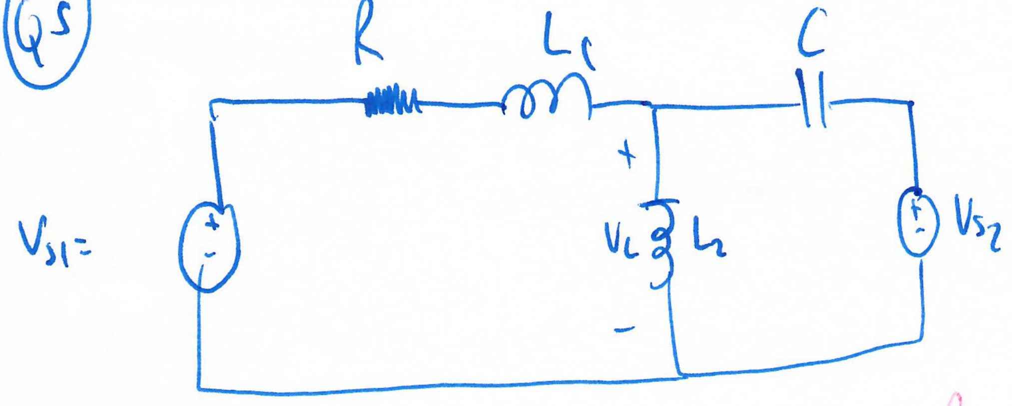
$K_3 = 1$

$K_2 = 4$

$K_1 = 1 - K_2$

$K_1 = 1 - 4 = -3$

Q5



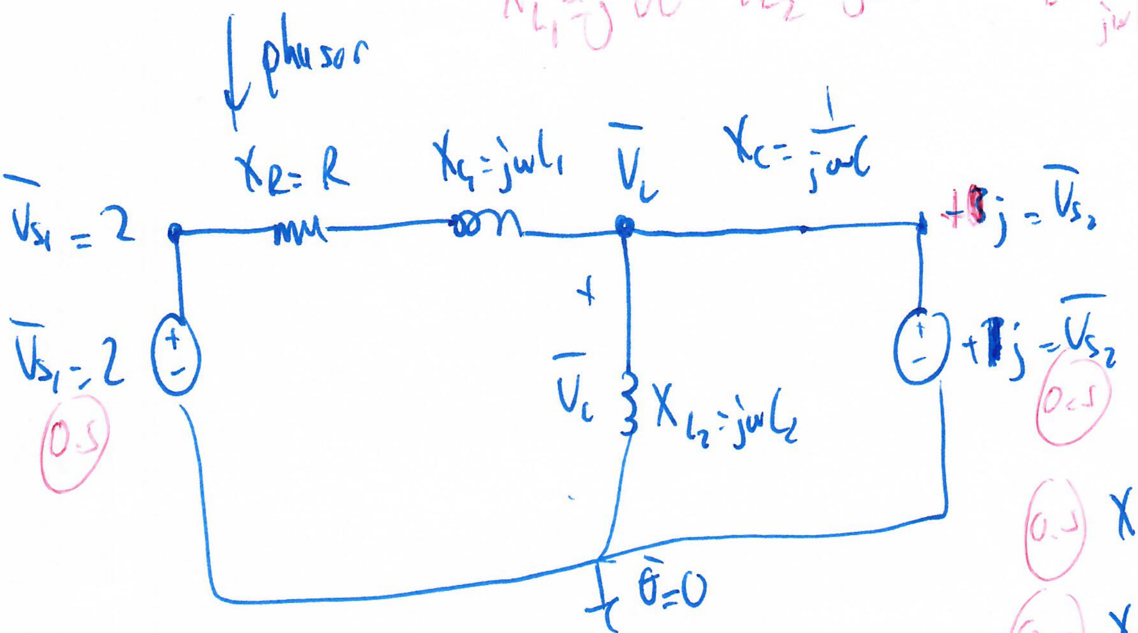
$V_{s_1} = 2\cos(t) \rightarrow \omega = 1 \frac{rad}{sec}$

$V_{s_2} = \cos(t + 90°)$

$\overline{V}_{s_1} = 2$ Volt

$\overline{V}_{s_2} = 3e^{-90j} = -3j$ Volt

$X_{L_1} = j\,\Omega$ $\quad X_{L_2} = j\,\Omega$ $\quad X_C = \frac{1}{j\omega C} = -j$



$R = 2$

$L_1 = L_2 = 1$ H

$C = 1$F

0.5 $X_R = R = 2\,\Omega$

0.5 $X_{L_1} = j\omega L_1 = j$

0.5 $X_{L_2} = j\omega L_2 = j$

0.5 $X_C = \frac{1}{j\omega C} = -j$

$$\frac{\overline{V}_{s_1} - \overline{V}_L}{R + j\omega L_1} + \frac{\overline{V}_{s_2} - \overline{V}_L}{\frac{1}{j\omega C}} = \frac{\overline{V}_L}{j\omega L_2}$$

$$\frac{2 - \overline{V}_L}{2 + j} + \frac{+j - \overline{V}_L}{-j} = \frac{\overline{V}_L}{j}$$ 4

$$\frac{2 - \overline{V}_L}{2 + j} + \frac{\overline{V}_L}{j} = \frac{\overline{V}_L}{j}$$

$$\frac{2 - \overline{V}_L}{2 + j} = +1$$

$$2 - \overline{V}_L = 2 + j$$

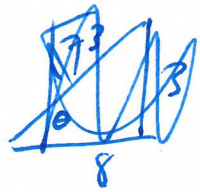
$$\overline{V}_L = -j = e^{-j90°}$$ 2

1

$V_{L_{sss}} = \cos(t - 90°)$

$V_{L_{sss}} = \sin(t)$ Volt

$\bar{V}_L = -j$

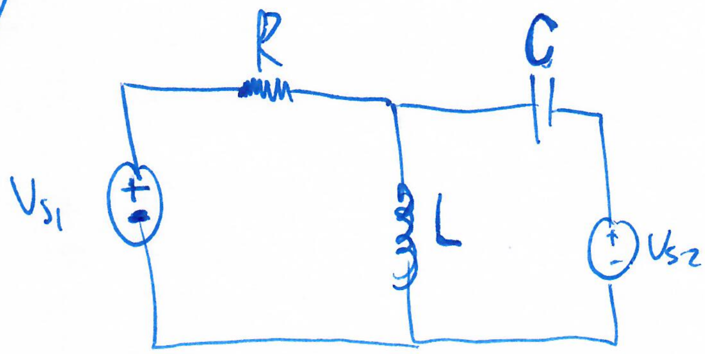


$\bar{V}_L = -j = e^{-j90}$

$V_{L_{sss}}(t) = \cos(t - 90^\circ)$

$V_{L_{sss}} = \sin(t)$ Volt

Q6 $R = 2$ $C = 1F$ $L = 1H$



$V_{s_1} = \cos(t)$ Volt

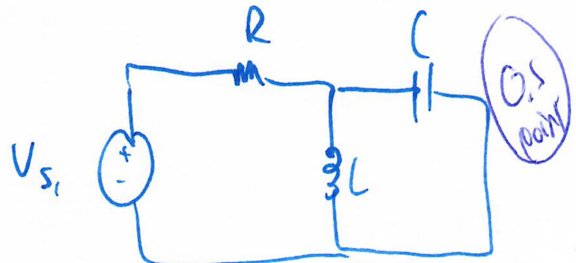
$V_{s_2} = \sin(2t)$ Volt

$X_R = R = 2\,\Omega$

$X_L = j\omega_1 L = j\,\Omega$

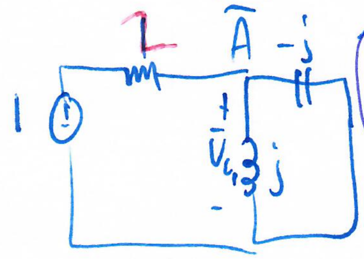
$X_C = \frac{1}{j\omega_1 C} = -j\,\Omega$

Kill V_{s_2} ($V_{s_1} = \cos(t)$ Volt) $\omega_1 = 1\ \frac{rad}{s}$



0.5 point

phaser $\longrightarrow$



2 point

2 point

0.5 point

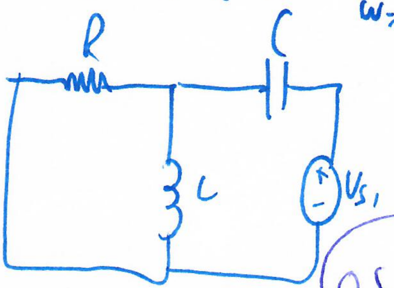
$$\frac{1-A}{2} = \frac{A}{j} + \frac{A}{-j}$$

$\bar{A} = 1$ $\bar{V}_{L_1} = \bar{A} = 1$ $V_{L_1} = \cos(t)$

1 point

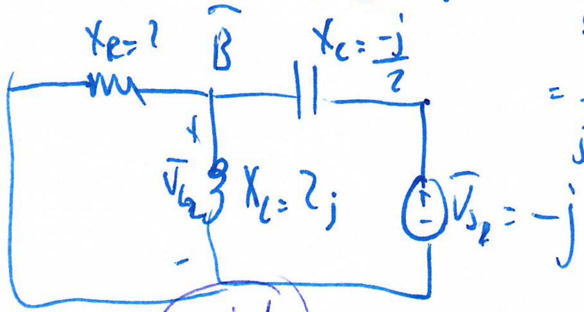
Kill V_{s_1} ($V_{s_2} = \sin(2t)$) $\omega_2 = 2\ \frac{rad}{sec}$

$X_R = 2\,\Omega = R$ $X_L = j\omega_2 L = 2j$ $X_C = \frac{1}{j\omega_2 C} = \frac{1}{j \times 2 \times 1}$



phasor $\longrightarrow$

0.5 point



2 point

$\bar{B} = \bar{V}_{L_2}$

$$\frac{\bar{V}_{s_1} - \bar{B}}{X_C} = \frac{\bar{B}}{X_L} + \frac{\bar{B}}{X_R}$$

2 point

$$\frac{-j - \bar{B}}{\frac{1}{2j}} = \frac{\bar{B}}{2j} + \frac{\bar{B}}{2}$$

$4j + 4\bar{B} = \bar{B} + \bar{B}j$

$$+2 - 2j\bar{B} = \frac{\bar{B}(1+j)}{2j}$$

$\cancel{4 - 4j\bar{B} = \bar{B} + \bar{B}j}$

$\cancel{4 = \bar{B} + 5\bar{B}j}$

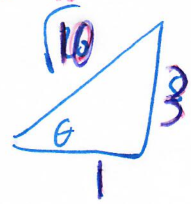
$4j = \bar{B}(j-3)$

$$\bar{B} = \frac{4j}{j-3}$$

$$\cancel{\frac{4}{1+5\bar{B}j} = \bar{B}} \quad \cancel{\frac{4(1-5j)}{\sqrt{26}} = \bar{B}} \quad \cancel{\frac{4}{\sqrt{26}}\left(\frac{1}{\sqrt{26}} - \frac{5}{\sqrt{26}}j\right) = \bar{B}}$$

$$\bar{B} = \frac{-4j}{3-j} = \frac{-4j(3+j)}{10} = \frac{4-12j}{10} = \frac{4}{10}(1-3j) = \frac{4}{\sqrt{10}}\left(\frac{1-3j}{\sqrt{10}}\right) = \bar{B}$$

$$\cancel{\bar{B} = \frac{4}{\sqrt{26}}(\cos\theta - j\sin\theta)}$$



$\theta = \arctan\frac{3}{1}$

$$\bar{B} = \frac{4}{\sqrt{10}} e^{-j\theta} \longrightarrow B = V_{L_2}(t) = \frac{4}{\sqrt{10}} \cos\left(2t - \arctan\frac{3}{1}\right)$$

0.5 point

1 point

~~1 point~~

$V_L(t) = V_{L_1} + V_{L_2}$

$$V_L(t) = \cos(t) + \frac{4}{\sqrt{10}} \cos\left(2t - \arctan\frac{3}{1}\right) \quad \text{Volt}$$

1 point