ECE 233 Make-up
17-01-2014

Q-1- $r(t)$ is the ramp function. Draw $k(t)=r(t)+r(t-1)-2r(t-2)-2r(t-3)+r(t-4)+r(t-5)$ with full details. (20 points)

Q-2- The differential equation governing the circuit variable $I_L(t)$ is given by

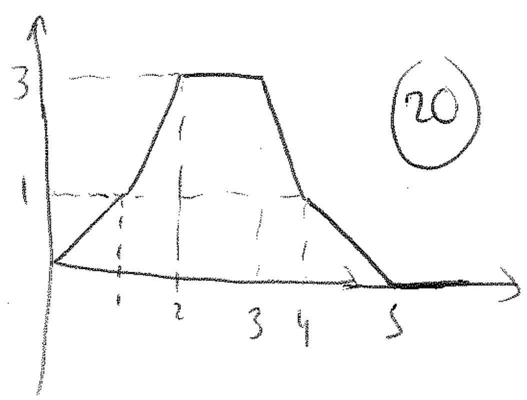
$$\frac{d^4 I_L}{dt^4} - I_L = Sin(2t)$$

Find the particular solution of this differential equation. (30 points)

Q-3- Design a second order circuit with two resistors and two capacitors and a voltage source V_s which has the following state space representation. (50 points)

$$\begin{bmatrix} \frac{dV_{C1}}{dt} \\ \frac{dV_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 0.5 & -0.5 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} V_S$$

Q-1 20



Q-2 $\frac{d^4 I_L}{dt^4} - I_L = Sin(2t)$ char-equation $s^4-1=0$

$(s^2+1)(s^2-1)$

$s_1=j$ $s_2=-j$ $s_3=1$ $s_4=-1$ 5

$s_1 \neq jw$ $w=2\ \frac{rad}{sec}$ hence $I_{Lp}=K_1 Sin(2t)+K_2 Cos(2t)$

$s_1 \neq jw$

$s_3 \neq jw$ 3 $\frac{dI_{Lp}}{dt} = 2K_1 Cos(2t) - 2K_2 Sin(2t)$ 4

$s_4 = jw$ $\frac{d^2 I_{Lp}}{dt^2} = -4K_1 Sin(2t) - 4K_2 Cos(2t)$ 4

$$\frac{d^3 I_{cp}}{dt^3} = -8K_1 \cos(2t) + 8K_2 \sin(2t)$$ (4)

$$\frac{d^4 I_{cp}}{dt^4} = 16K_1 \sin(2t) + 16K_2 \cos(2t)$$ (4)

$$\frac{d^4 I_{cp}}{dt^4} - I_{cp} = \sin(2t)$$

$$16K_1 \sin(2t) + 16K_2 \cos(2t) - \left[K_1 \sin(2t) + K_2 \cos(2t)\right] = \sin(2t)$$

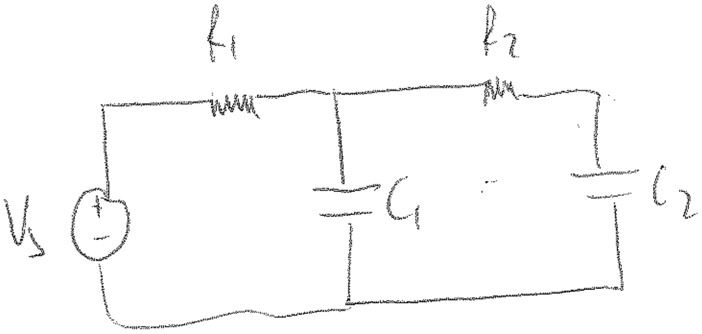
(5) $$15K_1 \sin(2t) + 15K_2 \cos(2t) = \sin(2t)$$

$$K_2 = 0 \quad K_1 = \frac{1}{15}$$

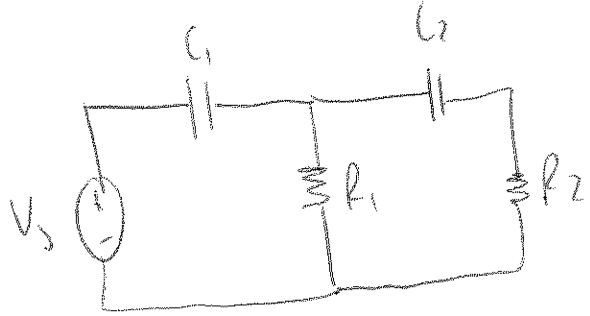
(6)

$$I_{cp} = \frac{1}{15} \sin(2t)$$

Q3 - We have a voltage source so the circuit configuration can be



or



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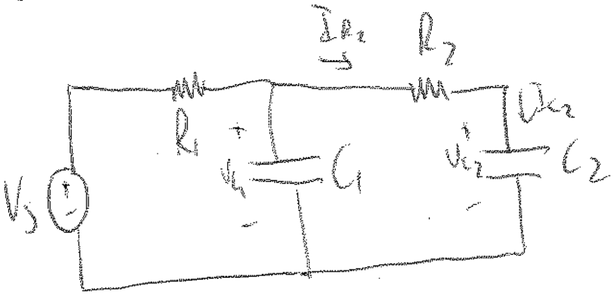

$V_s = V_{R_1} + V_{C_1}$

$I_{R_2} = I_{C_2} = C_2 \frac{dV_{C_2}}{dt}$

$V_{C_1} = V_{R_2} + V_{C_2}$

$V_{C_1} = R_2\left[C_2 \frac{dV_{C_2}}{dt}\right] + V_{C_2}$

$$\boxed{\frac{dV_{C_2}}{dt} = \frac{1}{R_2C_2} V_{C_1} - \frac{1}{R_2C_2} V_{C_2}}$$

Hence $\frac{1}{R_2C_2} = 0.5$ //

$V_s = V_{R_1} + V_{C_1}$ $\quad V_s = R_1\left[I_{C_1} + I_{C_2}\right] + V_{C_1}$

$$V_s = R_1\left[C_1\frac{dV_{C_1}}{dt} + C_2\frac{dV_{C_2}}{dt}\right] + V_{C_1}$$

$$V_s = R_1\left[C_1\frac{dV_{C_1}}{dt} + C_2\left(\frac{V_{C_1}}{R_2C_2} - \frac{V_{C_2}}{R_2C_2}\right)\right] + V_{C_1}$$

$$V_s = R_1C_1\frac{dV_{C_1}}{dt} + \left(\frac{R_1}{R_2} + 1\right)V_{C_1} + \frac{R_1}{R_2} V_{C_2}$$

$$\boxed{\frac{dV_{C_1}}{dt} = \frac{V_s}{R_1C_1} + \frac{1}{C_1R_2} V_{C_2} - \frac{R_1+R_2}{R_1C_1R_2} V_{C_1}}$$

Hence

$$\frac{1}{R_1C_1}=1 \qquad \frac{1}{C_1R_2}=1 \qquad \frac{R_1+R_2}{R_1C_1R_2}=2$$

$$\frac{1}{R_2C_2}=0.5$$

$R_1=1 \quad R_2=1$ $\Rightarrow$ All conditions are satisfied

$C_1=1 \quad C_2=2$

Hence circuit