Laboratory Experiment 4
Loading Effect and Wheatson Bridge

Loading Effect: A voltage divider circuit (shown in Figure 1) is used in order to divide the applied potential from a battery or a DC power supply. The applied potential is divided between the resistors (R_1 and R_2) in proportion to the resistance values. If a resistive load R_L is connected in parallel with one of the resistors of the voltage divider circuit (in Figure 2, the load resistance R_L and R_2 are connected in parallel), eventually we will observe a decrease in the potential portion where parallel connection is made. Let's find the voltages over R_1 and R_2 and total source current when there is no parallel connection (as in Figure 1) and when there is a load R_L connected in parallel with R_2 (as in Figure 2).

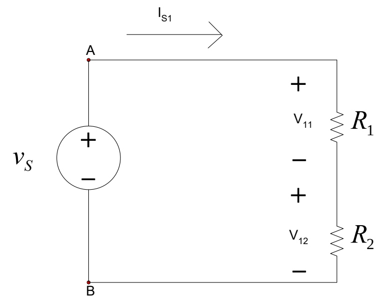

Figure 1: Voltage divider circuit.

In Figure 1:

$$V_{11} = \frac{R_1}{R_1 + R_2} V_S \text{ , } V_{12} = \frac{R_2}{R_1 + R_2} V_S \text{ and } I_{S1} = \frac{1}{R_1 + R_2} V_S$$

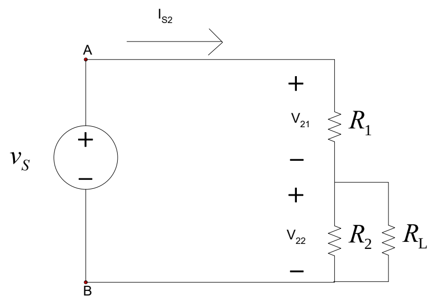

Figure 2: Voltage divider circuit with extra load resistor.

In Figure 2:

$$V_{21} = \frac{R_1 R_L + R_1 R_2}{R_1 R_L + R_2 R_L + R_1 R_2} V_S \text{ , } V_{22} = \frac{R_2 R_L}{R_1 R_L + R_2 R_L + R_1 R_2} V_S \text{ and}$$

$$I_{S2} = \frac{R_L + R_2}{R_1 R_L + R_2 R_L + R_1 R_2} V_S$$

From these equations we can make some comparisons,

$$V_{11} < V_{21} \text{ , } V_{12} > V_{22} \text{ and } I_{S1} \le I_{S2}$$

As seen from the inequalities above, the parallel connection of the load R_L to R_2 decreases the potential over R_2 and increases the potential over R_1 and also due to this parallel connection the overall source current value increases. The drop of potential due to the load connected in parallel is known as **loading effect**.

When we want to measure the potential difference between any two points we use voltmeters. Hence at each situation where voltmeter is used, we made a parallel connection to some circuit portion. This parallel connection causes a kind o voltage drop over that circuit portion. Generally Voltmeters have very high internal resistance values (a perfect voltmeter has infinite resistance). Hence loading effect is not so apparent at most of the cases. However the resistance of a voltmeter might change due to the voltage range selected and due to the type of the voltmeter (analog or digital). Loading effect is much more obvious when we use an analog multimeter since its resistance values are much lower compared to digital multimeter when we use the multimeters as voltmeters. If the resistors in the circuit have small resistances, the loading effect when we connect the analog multimeter (as voltmeter) is negligible and not much deviation is observed from the expected voltage measurement values. However if these resistors resistance values are relatively high (compared to the internal resistance of the analog mutimeter which is used as a voltmeter), the voltage measurements will deviate from the expected results in higher amounts because of the loading effect (as one can understand if R_2 has a comparable value with respect to R_L, the loading effect will be much more apparent.

1- Construct the circuits in Figure 3 with different R_1 and R_2 settings that are shown at each row of Table 1. First use the analog multimeter as the voltmeter (as shown in Figure 3) and **measure** the voltage over the resistor R_2 at different resistor settings. Secondly use the digital multimeter as the voltmeter (as shown in Figure 3) and **measure** the voltage over the resistor R_2 at different resistor settings. Finally, analytically solve the circuit in Figure 3 and calculate the voltage drop over the resistor R_2 that actually should be.

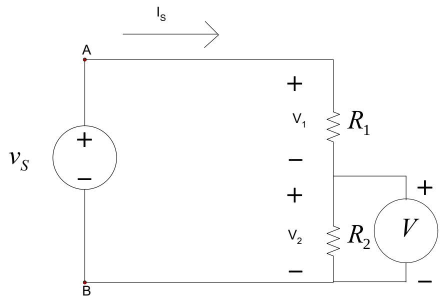

Figure 3: A voltage divider circuit with a voltmeter connected to second resistor.

Table 1: Voltage measurements and calculations over R_2

	R_1	R_2	Measured V_2 (by analog multimeter)	Measured V_2 (by digital multimeter)	Calculated V_2
1	1 kΩ	1 kΩ			
2	10 kΩ	10 kΩ			
3	100 kΩ	100 kΩ			
4	1 MΩ	1 MΩ			
5	10 MΩ	10 MΩ			

Questions:

- How can we understand that the digital multimeter has higher internal resistance compared to analog multimeter?
- Which multimeter is more confident in terms of resistance measurements?

Wheatstone bridge: Wheatstone bridge is a particular circuit which is used to measure unidentified resistance values. In order to figure out the functionality of this particular circuit, let's consider the circuit diagram shown in Figure 4. In this circuit there are five resistors. If no current is observed over the resistor R_5 (I_5=0 A), then this circuit is said to be in **balance condition**. What condition should be satisfied such that the Wheatstone bridge is balance?

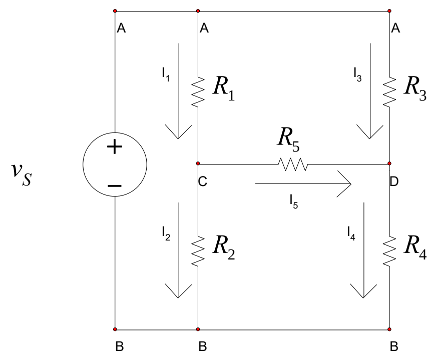

Figure 4: Wheatstone bridge circuit.

It is seen obvious that

$$I_1 = I_2 + I_5 \quad (1)$$

and

$$I_4 = I_3 + I_5 \quad (2)$$

If $I_5 = 0$ A (if the Wheatstone bridge is balanced),

$$I_1 = I_2 \quad (3)$$

and

$$I_4 = I_3 \quad (4)$$

Using (3) and (4) and applying node voltage method, we can write,

$$\frac{A-C}{R_1} = \frac{C-B}{R_2} \quad (5)$$

and

$$\frac{A-D}{R_3} = \frac{C-B}{R_4} \quad (6)$$

Again using node voltage method we can also write,

$$C = D + R_5 I_5 \quad (7)$$

As $I_5 = 0$ A, (7) can be simplified and we can obtain the relation

$$C = D \quad (8)$$

Using (8) inside (6), we can obtain

$$\frac{A-C}{R_3} = \frac{C-B}{R_4} \quad (9)$$

Now using (5) and (9), we can write

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad (10)$$

So the relation shown at (10) should be satisfied for balance condition.

2- Construct the circuit in Figure 5 according to different set of resistor values in Table 2. Measure the voltage over the voltmeter for each set of resistor values and according to your measurements indicate whether these set of resistors constitute a balanced Wheatstone bridge or not? Lastly, construct the circuit in Figure 6 and measure the resistance between nodes A and B (R_{AB}) for each set of resistors and record the on Table 2

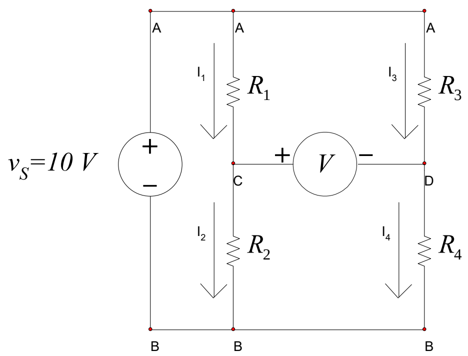

Figure 5: Wheatstone bridge circuit for checking balance condition and voltage measurements.

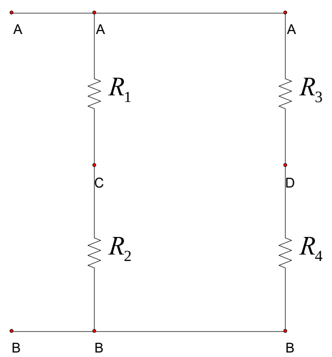

Figure 6: Wheatstone bridge circuit for resistance measurements.

Table 2: Voltage measurements over the Wheatstone bridge.

Resistor Set	R_1 (kΩ)	R_2 (kΩ)	R_3 (kΩ)	R_4 (kΩ)	V_{CD}	Balanced/ Unbalanced	R_{AB}
1.	1	1	1	1			
2.	1	10	1	10			
3.	10	1	10	1			
4.	1	1	10	10			
5.	1	10	10	1			

Questions:

- How many voltage dividers exist in the circuit shown in Figure 6?

3- Construct the Wheatstone bridge circuit in Figure 7. R_1=10 kΩ, R_2=5 kΩ, R_3=5 kΩ, R_{pot}= 10 kΩ (max value)

- Step 1: Adjust the potentiometer such that, the voltage over the multimeter is nearly equal to 0 (the Wheatstone bridge is balanced).
- Step 2: Take the potentiometer out of the circuit and measure its resistance value (R_{pot}).

- Step 3: Compare the potentiometer resistance value you have measured and the theoretical resistance value that should be in order to observe a balanced circuit.

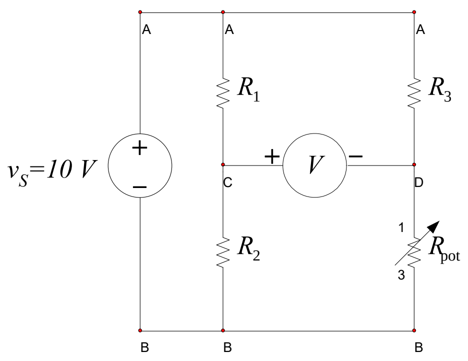

Figure 7: Wheatstone bridge circuit for resistance measurement over the potentiometer.