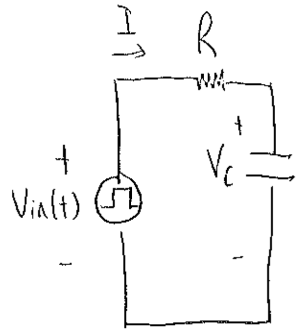
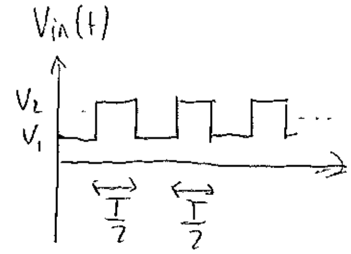
ECE 232

First Order Circuits (Preliminary Work)

1-a





$$V_{in} = RI + V_c \qquad V_{in} = RC\frac{dV_c}{dt} + V_c \qquad \frac{dV_c}{dt} + \frac{1}{RC}V_c = \frac{1}{RC}V_{in}$$

$$I = C\frac{dV_c}{dt}$$

$V_0 \rightarrow$ initial condition

Solution to the differential equation

$$V_c(t) = V_0\, e^{-(t-t_0)/\tau} + V_s\left(1 - e^{-(t-t_0)/\tau}\right)$$

where $\tau = RC$ (time constant), $V_0 \rightarrow$ initial condition, $V_s \rightarrow$ input, input either V_1 or V_2

For $0 < t < \frac{T}{2} \Rightarrow V_0 = 0$, $V_c(t) = V_2\left(1 - e^{-\frac{t}{\tau}}\right)$ $\quad V_c\left(\frac{T}{2}\right) = V_2\left(1 - e^{-\frac{T}{2\tau}}\right)$

For $\frac{T}{2} < t < T \Rightarrow V_0 = V_c\left(\frac{T}{2}\right)$, $V_{in}(t) = V_1$

$$V_c(t) = V_2\left(1 - e^{-\frac{T}{2\tau}}\right)e^{-(t-\frac{T}{2})/\tau} + V_1\left(1 - e^{-(t-\frac{T}{2})/\tau}\right)$$

For $T < t < \frac{3T}{2}$ $\quad V_0 = V_c(T)$, $V_{in}(t) = V_2$

$$V_c(t) = V_0\, e^{-(t-T)/\tau} + V_2\left(1 - e^{-(t-T)/\tau}\right)$$

$\vdots$

And so on for other time intervals

② if $T \gg \tau$ [the period of the square waveform is much greater than the time constant of RC circuit]

then, For $0<t<\frac{T}{2}$, $V_{in}(t) = V_2$

$$V_c(t) = V_2\left(1 - e^{-\frac{t}{\tau}}\right), \quad V_c\left(\frac{T}{2}\right) = V_2\left(1 - e^{-\frac{T}{2\tau}}\right) \approx V_2$$

For $\frac{T}{2}<t<T$, $V_{in}(t) = V_1$

$V_c\left(\frac{T}{2}\right) \approx V_2 = V_0$ [new initial condition when $\frac{T}{2}<t<T$] and

$$V_c(t) \approx V_2 \, e^{-(t-\frac{T}{2})/\tau} + V_1\left(1 - e^{-\frac{(t-\frac{T}{2})}{\tau}}\right)$$

$$V_c\left(\frac{T}{2}\right) \approx V_2 \qquad V_c(T) \approx V_1$$

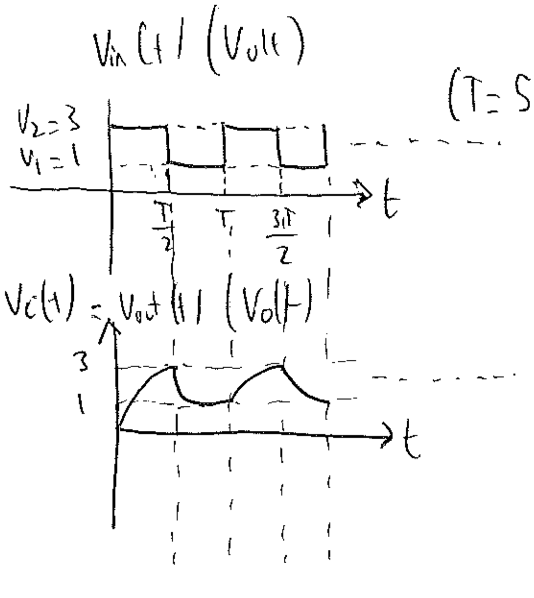
For $T<t<\frac{3T}{2}$, $V_{in}(t) = V_2$; initial condition $= V_c(T) \approx V_1 = V_0$

$$V_c(t) = V_1 \, e^{-(t-T)/\tau} + V_2\left(1 - e^{-(t-T)/\tau}\right)$$

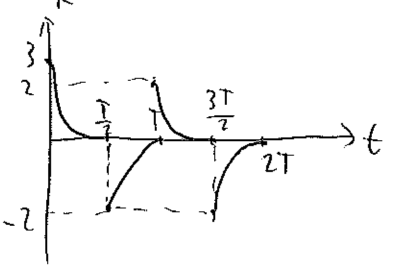
Hence when* $f = 2\,kHz$, $R = 3.3\,k\Omega$, $C = 4.7\,nF$

when $f = 2\,kHz \longrightarrow T = \frac{1}{2000} = 0.5\,ms = 500\,\mu s$ // $\quad \tau = 3.3\times10^3 \times 4.7\times10^{-9} = 15.51\,\mu s$

$T \gg \tau$



$V_R(t) = V_{in}(t) - V_c(t)$



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** when $f = 2kHz$; $R = 3.3k\Omega$, $C = 10nF$

$T = \frac{1}{2000} = 0.5ms = 500\mu s$ $\quad \tau = 3.3\times10^3 \times 10\times10^{-9} = 33\times10^{-6}$ sec

$= 33\ \mu sec$

$T \gg \tau$

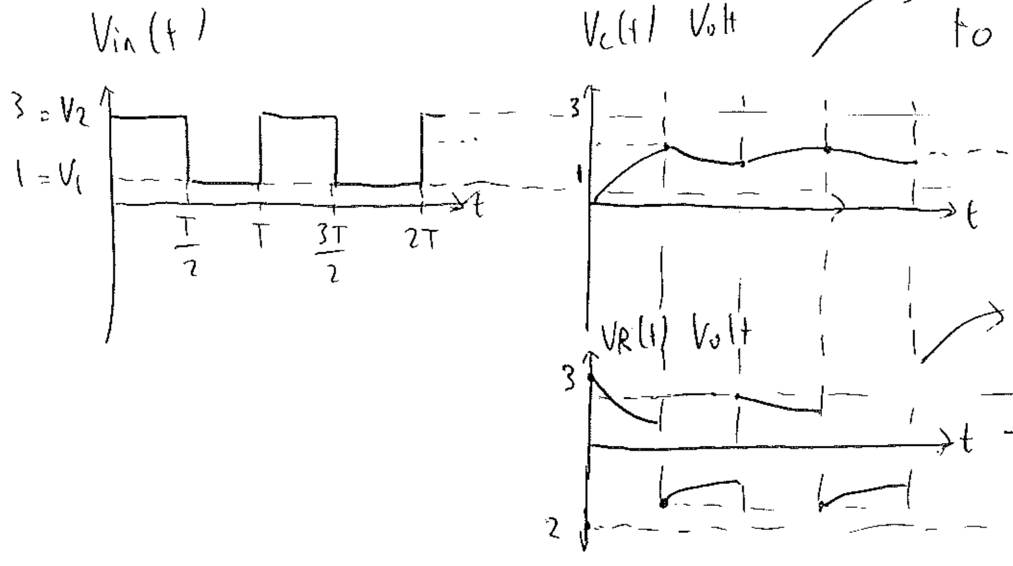
when we compare * and **, we can see that $\tau = 15.51\mu s$ for * but $\tau = 33\mu s$ for ** this means The first RC circuit "*" has a faster response.
But the waveforms obtained for V_C and V_R will be similar for "**"

*** when $f = 2kHz$, $R = 68k\Omega$, $C = 10nF$

$T = \frac{1}{2000} = 500\mu s$. $\quad \tau = 68\times10^3 \times 10\times10^{-9} = 680\mu sec$

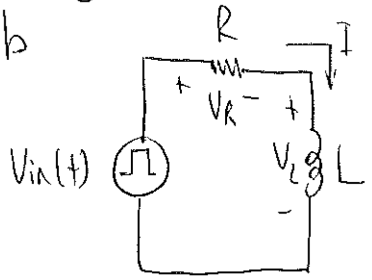
we cannot say that $T \gg \tau$, so this circuit has a very slow response. The positive and negative cycles of input waveform (square wave) will change suddenly before the capacitor is fully charged ($V_0 \not\approx V_2$) or decharged ($V_0 \not\approx V_1$). Hence the output wavefors are;

$V_C(t)$ will not reach to 3 or 1 volt



V_R will never increase or decrease to 0 Volt

1-b



$V_{in} = V_R + V_L \rightarrow V_{in} = RI + L\frac{dI}{dt}$

$V_L = L\frac{dI}{dt}$

$\frac{dI}{dt} + \frac{R}{L} I = \frac{V_{in}}{L}$

$V_{in} \rightarrow V_2$ or V_1

$\boxed{\tau = \frac{L}{R} \text{ time constant}}$

— if $0 < t < \frac{T}{2}$, $I(0) = 0 = I_0$, $V_{in} = V_2$

$$I(t) = \underbrace{I_0}_{0} e^{-\frac{t}{\tau}} + \frac{V_2}{R}\left[1 - e^{-\frac{t}{\tau}}\right] = \frac{V_2}{R}\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$V_R(t) = RI(t) = R\underbrace{I_0}_{0} e^{-\frac{t}{\tau}} + V_2\left[1 - e^{-\frac{t}{\tau}}\right] = V_2\left[1 - e^{-\frac{t}{\tau}}\right]$$

$$V_L = L\frac{dI_L}{dt} = L\frac{V_2}{R}\frac{d}{dt}\left[1 - e^{-\frac{t}{\tau}}\right] = \frac{V_2 L}{R}\frac{1}{\tau}e^{-\frac{t}{\tau}} = V_2 e^{-\frac{t}{\tau}}$$

$$I\left(\frac{T}{2}\right) = \frac{V_2}{R}\left[1 - e^{-\frac{T}{2\tau}}\right]$$

— if $\frac{T}{2} < t < T$, $I_0 = I\left(\frac{T}{2}\right) = \frac{V_2}{R}\left[1 - e^{-\frac{T}{2\tau}}\right]$, $V_{in} = V_1$

$$I(t) = I_0 e^{-\frac{(t-\frac{T}{2})}{\tau}} + \frac{V_1}{R}\left[1 - e^{-\frac{(t-\frac{T}{2})}{\tau}}\right]$$

$$V_R(t) = RI(t) = RI_0 e^{-\frac{(t-\frac{T}{2})}{\tau}} + V_1\left[1 - e^{-\frac{(t-\frac{T}{2})}{\tau}}\right]$$

$$V_L(t) = L\frac{dI(t)}{dt} = L\left[I_0\left(-\frac{1}{\tau}\right)e^{-\frac{(t-\frac{T}{2})}{\tau}}\right] + L\frac{V_1}{R}\left(\frac{1}{\tau}\right)e^{-\frac{(t-\frac{T}{2})}{\tau}}$$

$$= -RI_0 e^{-\frac{(t-\frac{T}{2})}{\tau}} + V_1 e^{-\frac{(t-\frac{T}{2})}{\tau}}$$

— if $T < t < \frac{3T}{2}$, $I_0 = I(T)$, $V_{in} = V_2$

$$I(t) = I_0 e^{-\frac{(t-T)}{\tau}} + \frac{V_2}{R}\left[1 - e^{-\frac{(t-T)}{\tau}}\right]$$

$$V_R(t) = RI_0 = RI_0 e^{-\frac{(t-T)}{\tau}} + V_2\left[1 - e^{-\frac{(t-T)}{\tau}}\right]$$

$$V_L = L\frac{dI}{dt} = -I_0 R\, e^{-\frac{(t-T)}{\tau}} + V_2\left[e^{-\frac{(t-T)}{\tau}}\right]$$

$\vdots$

and so on for other time intervals

Hence if $T \gg \tau$ (the period of the input waveform is much greater than the time constant of RL circuit $\tau = \frac{L}{R}$)

then <u>For $0 < t < \frac{T}{2}$</u>, $V_{in}(t) = V_2$, $I_0 = 0$

$$V_R(t) = V_2\left[1 - e^{-\frac{t}{\tau}}\right], \quad V_L(t) = V_2\, e^{-\frac{t}{\tau}} \quad \text{and} \quad I\left(\frac{T}{2}\right) \cong \frac{V_2}{R} = I_0$$

<u>For $\frac{T}{2} < t < T$</u>, $V_{in}(t) = V_1$, $I_0 = \frac{V_2}{R}$ //

$$V_R(t) = V_2\, e^{-\frac{(t-\frac{T}{2})}{\tau}} + V_1\left[1 - e^{-\frac{(t-\frac{T}{2})}{\tau}}\right]$$

$$V_L(t) = -V_2\, e^{-\frac{(t-\frac{t}{2})}{\tau}} + V_1\, e^{-\frac{(t-\frac{T}{2})}{\tau}} = \left[V_1 - V_2\right] e^{-\frac{(t-\frac{T}{2})}{\tau}}$$

<u>For $T < t < \frac{3T}{2}$</u>, $V_{in}(t) = V_2$, $I_0 = I(T) = \frac{V_1}{R}$ //

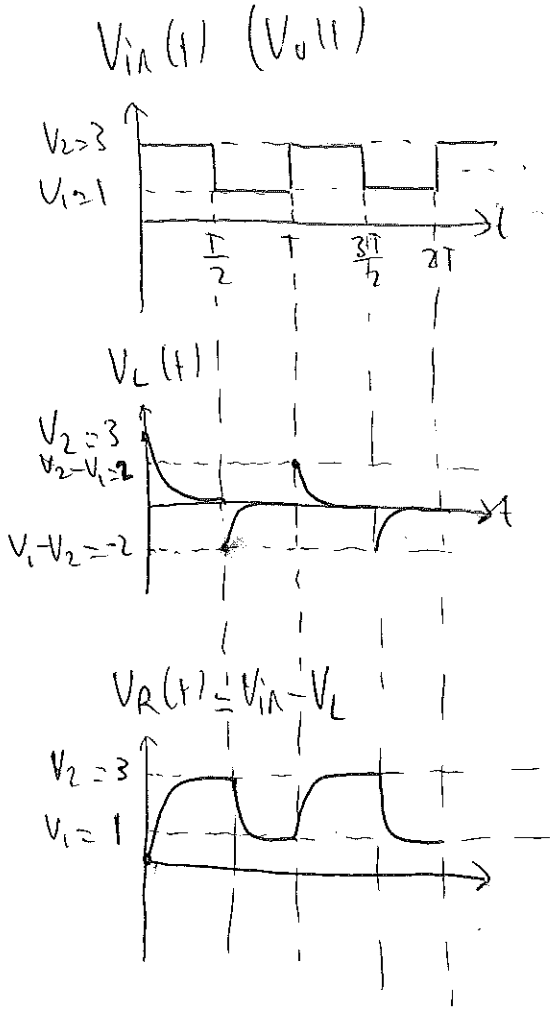
$$V_R(t) = V_1\, e^{-\frac{(t-T)}{\tau}} + V_2\left[1 - e^{-\frac{(t-T)}{\tau}}\right]$$

$$V_L(t) = -V_1\, e^{-\frac{(t-T)}{\tau}} + V_2\, e^{-\frac{(t-T)}{\tau}} = \left[V_2 - V_1\right] e^{-\frac{(t-T)}{\tau}}$$ //

* when $f = 2\,kHz$, $R = 3.3\,k\Omega$, $L = 0.1\,H$, $\tau = \frac{L}{R} = \frac{0.1}{3.3\times10^3} = 3.03\times10^{-5}\,sec = 30.3\,\mu sec$

$T = \frac{1}{2000} = 0.5\,msec = 500\,\mu sec$ $\quad T \gg \tau$ //

⑥



** When $f=2kHz$ $R=1.8k\Omega$

$L=0.1H$, $\tau=\frac{L}{R}=\frac{0.1}{1.8k\Omega}=55.5\mu sec$

$T \gg \tau$

where

$T=\frac{1}{2000}=500\mu sec$ //

Both * and ** are very fast RL circuits but * is faster. Hence V_L and V_R plots will be similar while * reaches to steady-state situation more rapidly.

*** $f=10kHz$ [a very rapid input waveform]

$T=\frac{1}{10kHz}=100\mu s$

$R=3.3$

$L=0.1$

$\tau=\frac{L}{R}=30.3\mu s$

For this circuit we can say that T and τ values are close to each other. Hence the inductor will always have some potential for positive and negative cycles of the input, it will never reach to steady-state short circuit position

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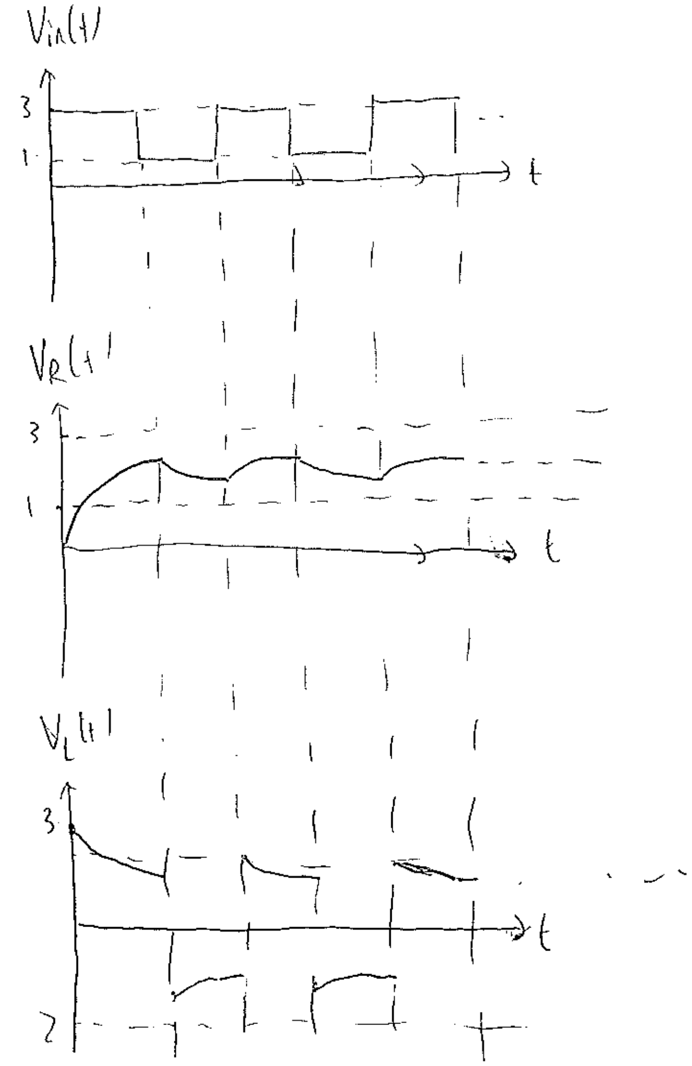
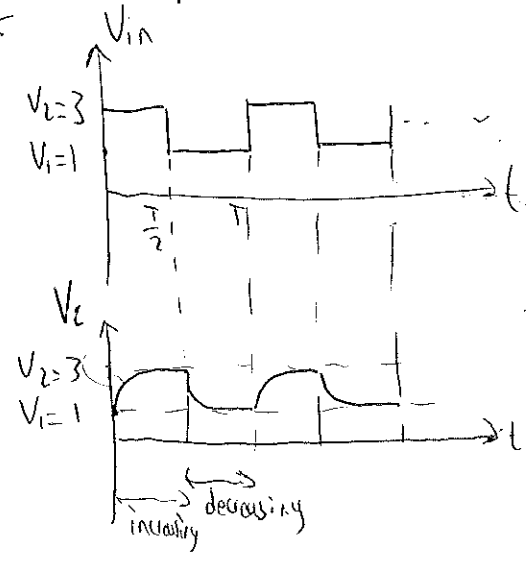


(c) The voltage waveform for RC circuit (Voltage over capacitor, V_c) is as follows (if the RC circuit is fast, or τ is small)

(1)*

(2)*



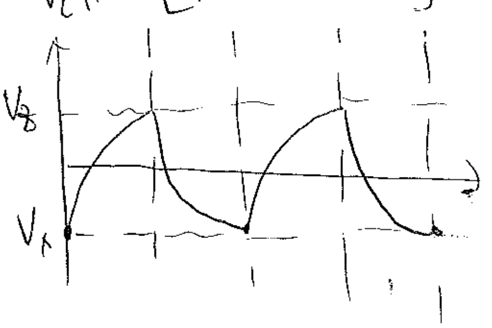
The V_c waveform can be written as

$$V_c(t) = V_2 + (V_A - V_2)\, e^{-\frac{t}{\tau}} \quad \text{(for increasing mode)}$$

$$V_c(t) = V_1 + (V_B - V_1)\, e^{-\frac{t}{\tau}} \quad \text{(for decreasing mode)}$$

Assuming that we see in oscilloscope in AC mode (for V_c)

(3)* $V_c(t)$ [in AC mode]



$V_A = V_x + V_Y$

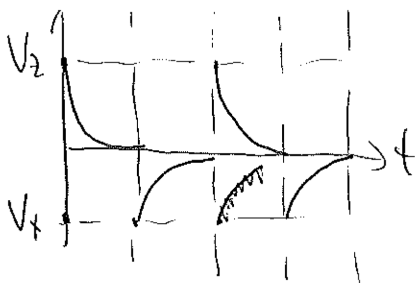
$V_B = V_z + V_Y$

where V_Y is the (DC) component in V_c.

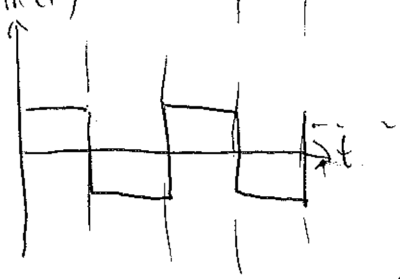
Than taking a point on graph (2)* in the increasing mode $\left[V_c(t) = V_2 + (V_A - V_2)\, e^{-\frac{t}{\tau}}\right]$ we can find τ directly.

for RL circuit, the voltage waveform "V_L" is as follows

$V_L(t)$ [in DC mode]



$V_{in}(t)$



Generally the DC level over the inductor decreases very rapidly to zero so $V_Y = 0$

$V_L(t) = K_1\, e^{-\frac{t}{\tau}}$ [for increasing mode]

$V_L(t) = K_2\, e^{-\frac{t}{\tau}}$ [for decreasing mode]

$K_2 < 0 < K_1$ $\qquad K_1 = V_z + V_Y \approx V_z$

$K_2 = V_x + V_Y \approx V_x$

Then taking a point for $V_L(t)$ graph in the increasing mode, we can find τ directly