ECE 233 final 17-01-2014

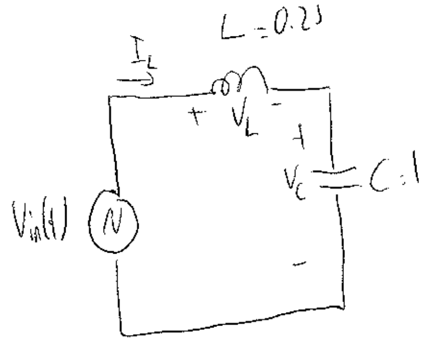
Q-1-

2 $X_L = j\omega L = j\omega L = j.1.1 = j\ \Omega$ $X_R = 1\ \Omega$ 2

2 $X_C = \frac{1}{j\omega C} = \frac{1}{j.1.1} = -j\ \Omega$ $V_{sp} = e^{j0} = e^{j0°} = \cos 0 + j\sin 0 = 1$ 2

$I_{Lp} = \frac{V_{sp}}{X_L + X_C + X_R} = \frac{1}{j1(-j)+1} = 1$ $I_{L_{sss}}(t) = 1\cos(t)$
5 2

2



$V_{in} = V_L + V_C$ $\qquad I_L = I_C = C\dfrac{dV_C}{dt}$ $\qquad \boxed{\dfrac{dV_C}{dt} = \dfrac{I_L}{1} = I_L}$

$V_{in} = L\dfrac{dI_L}{dt} + V_C$

$V_{in} = L\dfrac{d}{dt}\left[C\dfrac{dV_C}{dt}\right] + V_C \longrightarrow V_{in} = LC\dfrac{d^2V_C}{dt^2} + V_C$

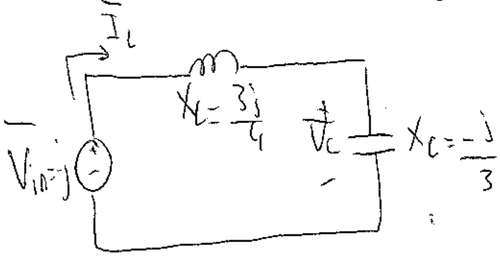
$V_{in}(t) = \sin(3t) \Rightarrow w = 3\,\frac{rad}{sec}$ $\qquad \dfrac{d^2V_C}{dt^2} + \dfrac{1}{LC}V_C = \dfrac{1}{LC}V_{in}$ $\qquad \dfrac{d^2V_C}{dt^2} + 4V_C = 4V_{in}$ 2

char-equation $s^2 + 4 = 0$ $\quad s_1 = 2j, \; s_2 = -2j$ 2 $\Rightarrow$ $s_1 \neq jw = 3j$, $s_1 \neq jw = 3j$ hence

(a) Sinusoidal steady state exists

(b) Use phasors $\quad V_{in}(t) = \sin(3t) = \cos(3t - 90°) \longrightarrow \overline{V_{in}} = e^{-j90} = \cos(-90) + j\sin(-90) = -j$ 1

$X_L = jwL = j \cdot 3 \cdot 0.25 = \dfrac{3j}{4}$ 1 $\qquad X_C = \dfrac{1}{jwC} = \dfrac{1}{j \cdot 3 \cdot 1} = -\dfrac{j}{3}$ 1



$\overline{I_L} = \dfrac{\overline{V_{in}}}{X_L + X_C} = \dfrac{-j}{\frac{3j}{4} - \frac{j}{3}} = \dfrac{-j}{\frac{9j - 4j}{12}} = \dfrac{-12j}{5j} = -\dfrac{12}{5}$ 3

$\overline{V_C} = \overline{I_L}\,X_C = -\dfrac{12}{5}\left(-\dfrac{j}{3}\right) = \dfrac{4}{5}j = \dfrac{4}{5}\left[0 + j\right] = \dfrac{4}{5}\left[\cos(90) + j\sin(90°)\right] = \dfrac{4}{5}e^{j90°}$

$V_{C_{sss}} = \dfrac{4}{5}\cos(3t + 90) = -\dfrac{4}{5}\sin(3t)$ 1

(c) $\dfrac{d^2V_C}{dt^2} + 4V_C = 4\sin(3t)$ $\qquad V_C(0) = 1\,\text{Volt}$

$\dfrac{dV_C}{dt}(0) = \dfrac{I_L(0)}{C} = \dfrac{0}{1} = 0\,\dfrac{\text{Volt}}{\text{sec}}$ 1

$V_{cp} = K_1\sin(3t) + K_2\cos(3t)$ $\qquad \boxed{\dfrac{d^2V_{cp}}{dt^2} = -9K_1\sin(3t) - 9K_2\cos(3t)}$ 1

$\dfrac{dV_{cp}}{dt} = 3K_1\cos(3t) - 3K_2\sin(3t)$ 1 1

$-9K_1\sin(3t) - 9K_2\cos(3t) + 4K_1\sin(3t) + 4K_2\cos(3t) = 4\sin(3t)$

1 $-5K_1\sin(3t) - 5K_2\cos(3t) = 4\sin(3t)$

$K_1 = -\frac{4}{5}$ $K_2 = 0$

$$\boxed{V_{cp} = -\frac{4}{5}\sin(3t)}$$ (1)

$s^2+4=0 \rightarrow$ char-equation

$V_{ch} = M_1\cos(2t) + M_2\sin(2t)$

$$V_c = V_{cp} + V_{ch} = -\frac{4}{5}\sin(3t) + M_1\cos(2t) + M_2\sin(2t)$$ (1)

$$V_c(0) = 1 = M_1$$

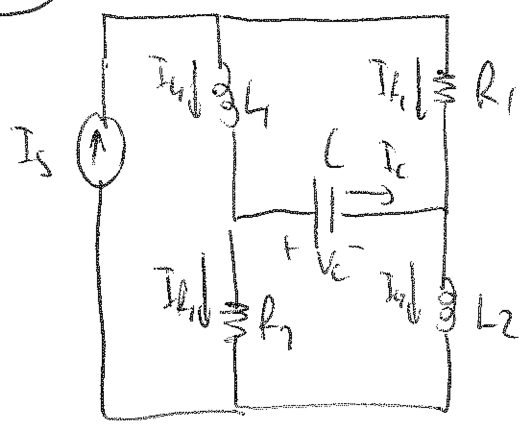
$$\frac{dV_c}{dt} = -\frac{12}{5}\cos(3t) - 2M_1\sin(2t) + 2M_2\cos(2t)$$ (1)

$$\left.\frac{dV_c}{dt}\right|_{t=0} = 0 = -\frac{12}{5} + 2M_2 \qquad M_2 = \frac{6}{5}$$

(2)

$$V_c(t) = -\frac{4}{5}\sin(3t) + \cos(2t) + \frac{6}{5}\sin(2t)$$

Q-3



$(I_{L_1} - I_c) = I_{R_2}$ 1

$I_{R_2} + I_{L_2} = I_s$ 1

$I_{L_1} - I_c + I_{L_2} = I_s \qquad I_{L_1} + I_{L_2} - I_s = I_c = C\dfrac{dV_c}{dt}$

$$\boxed{\frac{dV_c}{dt} = \frac{I_{L_1}}{C} + \frac{I_{L_2}}{C} - \frac{I_s}{C}} \quad 3 \qquad \boxed{I_C = I_{L_1} + I_{L_2} - I_s} \quad 1$$

$$V_{R_2} = V_c + V_{L_2} = V_c + L_2\frac{dI_{L_2}}{dt} = R_2 I_{R_2} = R_2(I_{L_1} + I_{L_2} - I_s)$$
$$= R_2(I_s - I_{L_2})$$

1 1 1

$$V_c + L_2\frac{dI_{L_2}}{dt} = R_2 I_{L_1} + R_2 I_{L_2} - R_2 I_s$$

$$\boxed{\frac{dI_{L_2}}{dt} = \frac{R_2}{L_2}I_{L_1} + \frac{R_2}{L_2}I_{L_2} - \frac{V_c}{L_2} - \frac{R_2}{L_2}I_s}$$

$$\boxed{\frac{dI_{L_2}}{dt} = \frac{R_2}{L_2}I_s - \frac{R_2}{L_2}I_{L_2} - \frac{V_c}{L_2}}$$

3

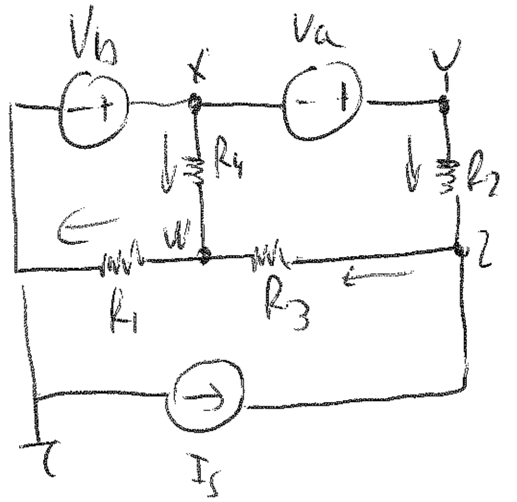
2 1

$$L_1\frac{dI_{L_1}}{dt} + V_c = V_{R_1} = I_{R_1}R_1 = R_1(I_s - I_{L_1})$$

$$\boxed{\frac{dI_{L_1}}{dt} = \frac{R_1}{L_1}I_s - \frac{R_1}{L}I_{L_1} - \frac{V_c}{L_1}} \quad 3$$

(2) $$\begin{bmatrix}\dot V_c\\ \dot I_{L_1}\\ \dot I_{L_2}\end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & \frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2}\end{bmatrix}\begin{bmatrix}V_c\\ I_{L_1}\\ I_{L_2}\end{bmatrix} + \begin{bmatrix}-\frac{1}{C}\\ \frac{R_1}{L_1}\\ \frac{R_2}{L_2}\end{bmatrix} I_s$$

(Q-4)



(1.5) (1.5)

(a) $\boxed{X = V_b}$, $\boxed{Y = V_a + V_b}$

(b) $I_s + \dfrac{Y - Z}{R_2} = \dfrac{Z - W}{R_3}$ (6)

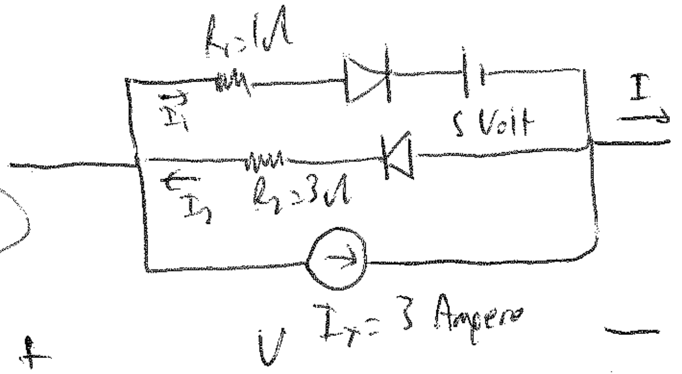
$$\boxed{I_s + \frac{V_a + V_b - Z}{R_2} = \frac{Z - W}{R_3}}$$

(6) $$\boxed{\frac{Z - W}{R_3} + \frac{X - W}{R_4} = \frac{W}{R_1}}$$

$$\boxed{\frac{Z - W}{R_3} + \frac{V_b - W}{R_4} = \frac{W}{R_1}}$$

(Q-5)

Gidiş 3/4



$\boxed{I = I_1 + I_x - I_2}$

Critical points 0, 5 Volt

* if $V > 5$ Volt $\quad I_1$ ✓ $\quad I_x$ ✓ $\quad I_2$ ✗

$I_1 = I_1 + I_x = I_1 + 3 \quad I_1 = (I - 3)$

$V = R_1 I_1 + 5 = 1(I-3) + 5$

$\boxed{V = I + 2} \rightarrow \boxed{I = V - 2}$ (2)

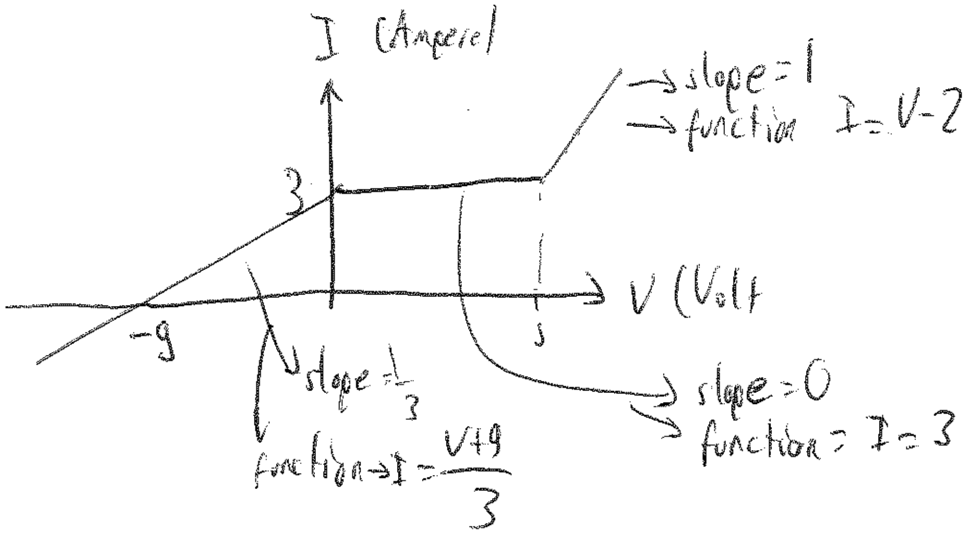
* if $0 < V < 5$ Volt $\quad I_1$ ✗ $\quad I_2$ ✗ $\quad I_x$ ✓ $\quad \boxed{I = I_x = 3}$ (3)

* if $V < 0$ $\quad I_1$ ✗ $\quad I_2$ ✓ $\quad I_x$ ✓ $\quad I = I_x - I_2$

$I_2 = I_x - I$
$= (3 - I)$

$V = -I_2 R_3$
$= -(3 - I)3 = 3I - 9$ (2)

$\boxed{V = 3I - 9} \quad I = \dfrac{V + 9}{3}$



1

Q-6 $\quad 0 < t < \ln 2$

Second capacitor is inactive hence $V_2(\ln 2^-) = V_2(0) = 2.5$ Volt 1

The functioning circuit

Gidiş 6

$V_1(0) = 1$ Volt 2

$$C_1 \frac{dV_1}{dt} + \frac{V_1}{R} = 0 \rightarrow \frac{dV_1}{dt} + \frac{1}{C_1 R} V_1 = 0$$

$$\frac{dV_1}{dt} + V_1 = 0 \quad \text{(only homogeneous solution)}$$

$$V_1(t) = K e^{-t} \quad 2$$

$$\rightarrow V_1(0) = 1 = K \qquad V_1(t) = e^{-t}$$

$$V_1(\ln 2^-) = e^{-\ln 2} = e^{\ln(\frac{1}{2})} = \frac{1}{2} \text{ Volt} \quad 3$$

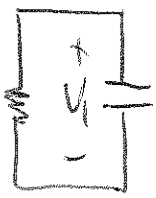
$t \geq \ln 2$ (preserve the amount of charges)

$$Q(\ln 2^-) = Q(\ln 2^+)$$

$$C_1 V_1(\ln 2^-) + C_2 V_2(\ln 2^-) = (C_1 + C_2) V_1(\ln 2^+) \quad 1$$

$$1 \times \frac{1}{2} + 3\,\frac{5}{2} = (1+3) V_c(\ln 2^+) \qquad V_c(\ln 2^+) = 2 \text{ Volt}$$

new functioning circuit 3



$1\Omega = R$ $\quad$ 1 $\quad C_T = C_1 + C_2 = 4F$

$V_c(\ln 2^+) = 2$ Volt

$$C_T \frac{dV_1}{dt} + \frac{V_1}{R} = 0$$

$$\frac{dV_1}{dt} + \frac{V_1}{4} = 0 \quad \text{(only homogeneous solution)}$$

$$V_1(t) = M e^{-\frac{(t - \ln 2)}{4}} \quad 1$$

$$V_1(\ln 2^+) = 2 = M$$

$$V_1(t) = 2\, e^{-\frac{(t - \ln 2)}{4}}$$

$0 < t < \ln 2 \quad V_1(t) = e^{-t}$

$t \geq \ln 2 \quad V_1(t) = 2e^{-\frac{(t-\ln 2)}{4}}$