ECE 233 final

Sayfa 1

2011–2012 fall

1 — $f(t) = -\frac{1}{3}e^{-2t} + \frac{1}{8}e^{-8t}$

1 $\boxed{f(0) = -\frac{1}{3} + \frac{1}{8} = \frac{-5}{24}}$

1 $\boxed{f'(0) = \frac{2}{3} - 1 = -\frac{1}{3}}$

1 $\boxed{f'(t) = \frac{2}{3}e^{-2t} - e^{-8t}}$

$f'(t^*) = 0 = \frac{2}{3}e^{-2t^*} - e^{-8t^*} = 0 \qquad \frac{2}{3}e^{-2t^*} = e^{-8t^*} \qquad e^{6t^*} = \frac{3}{2}$

$6t^* = \ln\left(\frac{3}{2}\right)$ 1 $t^* = \frac{1}{6}\ln\left(\frac{3}{2}\right)$ minimum

when $t = t^*$, $f(t^*) = -\frac{1}{3}e^{-2t^*} + \frac{1}{8}e^{-8t^*}$

$= -\frac{1}{3}e^{-2\frac{1}{6}\ln\left(\frac{3}{2}\right)} + \frac{1}{8}e^{-8\frac{1}{6}\ln\left(\frac{3}{2}\right)}$

$= -\frac{1}{3}e^{\ln\left(\frac{3}{2}\right)^{-\frac{1}{3}}} + \frac{1}{8}e^{\ln\left(\frac{3}{2}\right)^{-\frac{4}{3}}}$

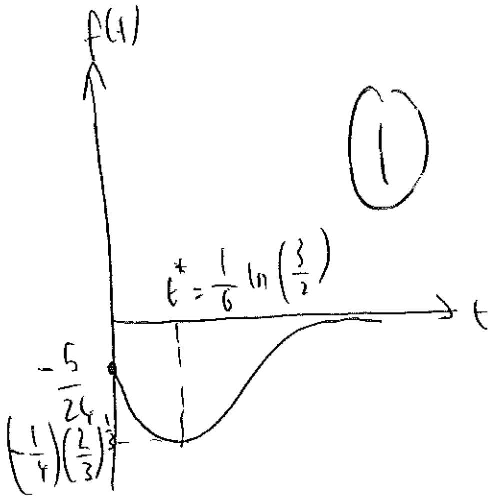
$= -\frac{1}{3}e^{\ln\left(\frac{2}{3}\right)^{\frac{1}{3}}} + \frac{1}{8}e^{\ln\left(\frac{2}{3}\right)^{\frac{4}{3}}}$

$= -\frac{1}{3}\left(\frac{2}{3}\right)^{\frac{1}{3}} + \frac{1}{8}\left(\frac{2}{3}\right)^{\frac{4}{3}}$

$= \left(\frac{2}{3}\right)^{\frac{1}{3}}\left[-\frac{1}{3} + \frac{1}{8}\left(\frac{2}{3}\right)\right] = \left(\frac{2}{3}\right)^{\frac{1}{3}}\left(-\frac{1}{3} + \frac{1}{12}\right)$

2 $= \left(\frac{2}{3}\right)^{\frac{1}{3}}\left(-\frac{1}{4}\right)$

$\boxed{\lim_{t \to \infty} f(t) = 0}$ 1



1

$f(t_x) = 0 = -\frac{1}{3}e^{-2t_x} + \frac{1}{8}e^{-8t_x}$

$\frac{1}{3}e^{-2t_x} = \frac{1}{8}e^{-8t_x}$

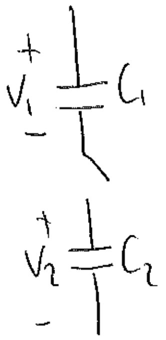
$e^{6t_x} = \frac{3}{8} \qquad 6t_x = \ln(3) - \ln(8)$

$t_x = \frac{\ln(3) - \ln(8)}{6} \rightarrow$ so $t_x < 0$

2

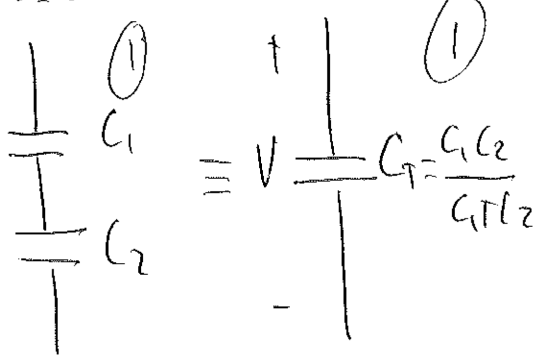
Sayfa 2

2- $t=0^-$ $t=0^+$



$E_{initial\_C_1} = \frac{1}{2} C_1 V_1^2(0^-)$

$E_{initial\_C_2} = \frac{1}{2} C_2 V_2^2(0^-)$



1

$V_1(0^-)+V_2(0^-) = V(0^+) =$

Prove that

$E_{initial\_C_1} + E_{initial\_C_2} \overset{?}{\geq} E_{final}$

$E_{final} = \frac{1}{2} C_T V^2(0^+)$

$= \frac{1}{2} \frac{C_1 C_2}{C_1+C_2} \left[V_1(0^-)+V_2(0^-) \right]^2$

Let $V_1(0^-) = a$ $V_2(0^-) = b$

$\frac{1}{2} C_1 a^2 + \frac{1}{2} C_2 b^2 \overset{?}{\geq} \frac{1}{2} \frac{C_1 C_2}{C_1+C_2} [a+b]^2$ 3 ifadeyi açmak

$\frac{1}{2} C_1^2 a^2 + \frac{1}{2} C_1 C_2 a^2 + \frac{1}{2} C_1 C_2 b^2 + \frac{1}{2} C_2^2 b^2 \overset{?}{\geq} \frac{1}{2} \left[a^2 C_1 C_2 + 2ab C_1 C_2 + b^2 C_1 C_2 \right]$

$\frac{1}{2} C_1^2 a^2 - ab C_1 C_2 + \frac{1}{2} C_2^2 b^2 \overset{?}{\geq} 0$ $\frac{1}{2}\left[(C_1 a - C_2 b)^2 \right] \geq 0$ always true since

2 bu biçime getirmek

1 tam kare ifadesi

$(C_1 a - C_2 b)^2$ is always positive

Sayfa 3

3- $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = \sin(t)$

ⓐ $s^2+2s+1=0$ char equation (1)

ⓑ $s_{1,2} = -1$ (2)

ⓒ $x_h(t) = (A+Bt)e^{-t}$ // (2)

4- $\frac{d^2x}{dt^2} + 2x = e^{-t}$

ⓐ $s^2+2=0$ (1)

ⓑ $s_{1,2} = \mp j\sqrt{2}$ (2)

ⓒ $X_h(t) = A_1\cos(\sqrt{2}t) + A_2\sin(\sqrt{2}t)$ (2)

5- $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2 = \frac{1}{t}$

ⓐ $s^2-2s+2=0$ (1)

ⓑ $s_{1,2} = 1\mp j$ (2)

ⓒ $X_h(t) = e^{t}\left[A_1\cos(t) + A_2\sin(t)\right]$ (2)

6- $\frac{d^2x}{dt^2} + x = 4\sin(t)$

char equation $s^2+1=0$

$s_{1,2} = \mp j \rightarrow$ natural frequencies

input $4\sin(t)$

↳ angular frequency $= w = 1\,\frac{rad}{sn}$

Hence $\boxed{s_1 = j = j \cdot w = j}$

$X_p(t) = t\left[A_1\cos(t) + A_2\sin(t)\right]$

$\frac{dx_p}{dt} = \left[A_1\cos(t) + A_2\sin(t)\right] + t\left[-A_1\sin(t) + A_2\cos(t)\right]$ (3)

$\frac{d^2x_p}{dt^2} = \left[-A_1\sin(t) + A_2\cos(t)\right] + \left[-A_1\sin(t) + A_2\cos(t)\right] + t\left[-A_1\cos(t) + A_2\sin(t)\right]$

$= -2A_1\sin(t) + 2A_2\cos(t) - t\left[A_1\cos(t) + A_2\sin(t)\right]$ (5)

(3)

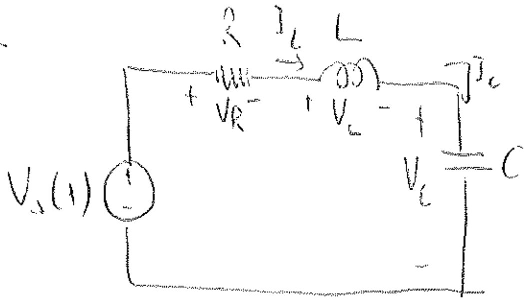
$-2A_1\sin(t) + 2A_2\cos(t) - t\left[A_1\cos(t) + A_2\sin(t)\right] + t\left[A_1\cos(t) + A_2\sin(t)\right] = 4\sin(t)$

$-2A_1\sin(t) + 2A_2\cos(t) = 4\sin(t)$ $A_2 = 0$, $A_1 = -2$

$x_p(t) = t\left[-2\cos(t)\right]$ (1)

Sayfa 4

7 —



2

$$C\frac{dV_c}{dt} = I_c = I_L \qquad 2 \qquad \boxed{\frac{dV_c}{dt} = \frac{I_L}{C}}$$

2 1 1

$$V_s = V_R + V_L + V_C \qquad V_s = RI_L + L\frac{dI_L}{dt} + V_C$$

$$\boxed{\frac{dI_L}{dt} = \frac{V_s}{L} - \frac{R}{L}I_L - \frac{V_c}{L}}$$

3

* $$\frac{d^2V_c}{dt^2} = \frac{1}{C}\frac{dI_L}{dt} = \frac{1}{C}\left[\frac{V_s}{L} - \frac{R}{L}I_L - \frac{1}{L}V_c\right] = \frac{V_s}{LC} - \frac{R}{LC}I_L - \frac{1}{LC}V_c$$

$$\frac{d^2V_c}{dt^2} + \frac{V_c}{LC} = \frac{V_s}{LC} - \frac{R}{L}\frac{dV_c}{dt} \qquad \boxed{\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{1}{LC}V_s}$$

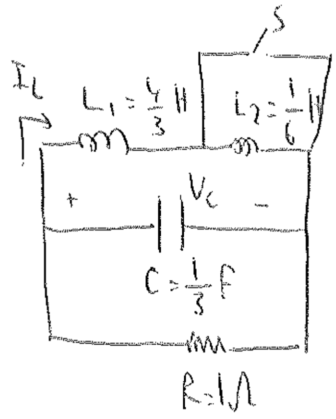
2 2

* $$\frac{d^2I_L}{dt^2} = \frac{1}{L}\frac{dV_s}{dt} - \frac{R}{L}\frac{dI_L}{dt} - \frac{1}{L}\frac{dV_c}{dt}$$

$$\frac{d^2I_L}{dt^2} + \frac{R}{L}\frac{dI_L}{dt} = \frac{1}{L}\frac{dV_s}{dt} - \frac{I_L}{LC} \rightarrow \boxed{\frac{d^2I_L}{dt^2} + \frac{R}{L}\frac{dI_L}{dt} + \frac{1}{LC}I_L = \frac{1}{L}\frac{dV_s}{dt}}$$

2

8-



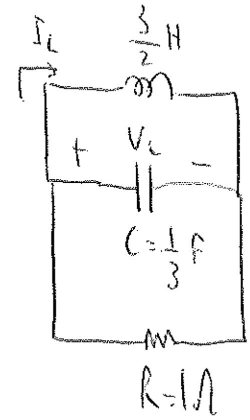
$V_c(0) = 5$ Volt

$I_L(0) = 0$ Amper

$0 < t < \ln(2)$ swich open (L_1 and L_2 series)

$$L_T = L_1 + L_2 = \frac{3}{2} \text{ H}$$

1



$V_c(0) = 5$ Volt

$I_L(0) = 0$ Amper

$$\frac{V_c}{R} + I_L + C\frac{dV_c}{dt} = 0 \qquad L_T\frac{dI_L}{dt} = V_c$$

$$V_c + I_L + \frac{1}{3}\frac{dV_c}{dt} = 0 \qquad \frac{3}{2}\frac{dI_L}{dt} = V_c$$

(1) $\boxed{\frac{dV_c}{dt} = -3V_c - 3I_L}$ (2) $\boxed{\frac{dI_L}{dt} = \frac{2}{3}V_c}$

$$* \frac{d^2 I_L}{dt^2} = \frac{2}{3}\frac{dV_c}{dt} = \frac{2}{3}\left[-3V_c - 3I_L\right] = -2V_c - 2I_L = -\frac{3}{2}\times 2\frac{dI_L}{dt} - 2I_L$$

Hence $\frac{d^2 I_L}{dt^2} + 3\frac{dI_L}{dt} + 2I_L = 0$ (3)

$I_L(0) = 0$ Amper (0.5)

$\frac{dI_L}{dt}(0) = \frac{2}{3}V_c(0) = \frac{10}{3}$

char-equation: $s^2 + 3s + 2 = 0$ (1)

$s_{1,2} = -1, -2$ (0.5) (natural frequencies) (overdamped case)

(1) $I_L(t) = A_1 e^{-t} + A_2 e^{-2t}$

$I_L(0) = 0 = A_1 + A_2$ (0.5)

$\frac{dI_L}{dt} = -A_1 e^{-t} - 2A_2 e^{-2t}$

$\frac{dI_L}{dt}(0) = \frac{10}{3} = -A_1 - 2A_2$ (0.5)

$A_1 = \frac{10}{3}$ $A_2 = -\frac{10}{3}$ (0.5)

Sayfa 6

$$I_L(t) = \frac{10}{3}e^{-t} - \frac{10}{3}e^{-2t} \qquad L_T\frac{dI_L}{dt} = V_L = V_C = \frac{3}{2}\frac{d}{dt}\left[-\frac{10}{3}e^{-2t} + \frac{10}{3}e^{-t}\right]$$

$$(1)\quad V_C(t) = -5e^{-t} + 10e^{-2t}$$

Hence when $0 \leq t < \ln(2) \rightarrow \begin{cases} V_C(t) = -5e^{-t} + 10e^{-2t} \\ I_L(t) = \frac{10}{3}e^{-t} - \frac{10}{3}e^{-2t} \end{cases}$

$$V_C(\ln(2)) = -5e^{-\ln(2)} + 10e^{-2\ln(2)} = -5e^{\ln(\frac{1}{2})} + 10e^{\ln(\frac{1}{4})} = -5\times\frac{1}{2} + 10\times\frac{1}{4} = 0 \quad (0.5)$$

$V_C(\ln(2^-)) = 0$ Volt

$$I_L(\ln(2^-)) = \frac{10}{3}e^{-\ln(2)} - \frac{10}{3}e^{-2\ln(2)} = \frac{10}{3}e^{\ln(\frac{1}{2})} - \frac{10}{3}e^{\ln(\frac{1}{4})} = \frac{10}{3}\times\frac{1}{2} - \frac{10}{3}\times\frac{1}{4} = \frac{5}{6} \text{ Amper} \quad (0.5)$$

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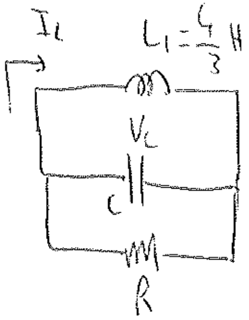
when $t > \ln(2)$ switch closed [L_2 shorted]



$\phi(\ln(2^-)) = \phi(\ln(2^+))$ [conserve total flux]

$I_L(\ln(2^-))\,L_T = L_1\,I_L(\ln(2^+))$ (1)

$\frac{5}{6} \times \frac{3}{2} = \frac{4}{3} I_L(\ln(2^+)) \qquad I_L(\ln(2^+)) = \frac{15}{16}$ Amper

(0.5)

$V_C(\ln(2^+)) = V_C(\ln(2^-)) = 0$ Volt (no change)

(To find the diff equations just put L_1 instead of L_T for the diff. equations valid for $t > \ln(2)$)

$$\frac{d^2I_L}{dt^2} + 3\frac{dI_L}{dt} + \frac{9}{4}I_L = 0 \qquad I_L(\ln(2^+)) = \frac{15}{16}$$

(2) (0.5)

$$\frac{dI_L}{dt}(\ln(2^+)) = \frac{3}{4}V_C(\ln(2^+)) = 0$$

(1) $s^2 + 3s + \frac{9}{4} = 0$ (char-equation) (0.5) $s_{1,2} = -\frac{3}{2}$ (natural frequencies) (critically-damped case)

Hence $I_L(t) = (At + B)e^{-\frac{3}{2}t}$ or $I_L(t) = [A(t - \ln(2)) + B]e^{-\frac{3}{2}(t - \ln(2))}$

(1)

Sayfa 7

$$\frac{dI_L}{dt} = A\, e^{-\frac{3}{2}(t-\ln(2))} + \left(-\frac{3}{2}\right) e^{-\frac{3}{2}(t-\ln(2))}\left[A(t-\ln 2)+B\right]$$ 1

$$I_L(\ln(2)) = B = \frac{15}{16}$$ 0.5

$$\frac{dI_L}{dt}(\ln(2^+)) = A - \frac{3}{2}B = 0 \rightarrow A = \frac{45}{32}$$ 0.5

$$I_L(t) = \left[\frac{45}{32}(t-\ln(2)) + \frac{15}{16}\right] e^{-\frac{3}{2}(t-\ln(2))}$$ 1 $\rightarrow$ when $t \geq \ln(2)$

$$L_1 \frac{dI_L}{dt} = V_0 = \frac{4}{3}\left[\frac{45}{32} e^{-\frac{3}{2}(t-\ln(2))} + \left(-\frac{3}{2}\right) e^{-\frac{3}{2}(t-\ln(2))}\left[\frac{45}{32}(t-\ln(2)) + \frac{15}{16}\right]\right]$$

1

$$V_L(t) = \frac{15}{8} e^{-\frac{3}{2}(t-\ln(2))} - 2\left[\frac{45}{32}(t-\ln(2)) + \frac{15}{16}\right]$$ $\rightarrow$ when $t \geq \ln(2)$

Gidiş 10 puan