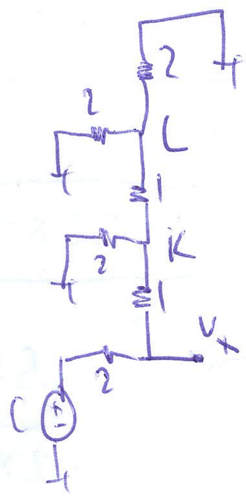
2010-2011 ECE 233 Midterm Solutions

Q-1 - Use superposition theorem

Kill V_1 and V_2



$$\frac{C-V_x}{2} = \frac{V_x-K}{1} \quad (3) \qquad \frac{C}{2} = 1.5V_x - K$$

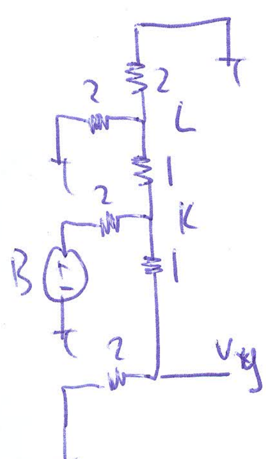
$$\frac{V_x-K}{1} = \frac{K}{2} + \frac{K-L}{1} \quad (3) \qquad V_x = 2.5K - L$$

$$\frac{K-L}{1} = \frac{L}{2} + \frac{L}{2} \quad (3) \rightarrow K = 2L \rightarrow V_x = 2.5K - \frac{K}{2}$$

$$V_x = 2K \rightarrow \frac{C}{2} = 1.5V_x - \frac{V_x}{2}$$

$$\boxed{\frac{C}{2} = V_x} \quad (4)$$

Kill V_1 and V_3



$$(3) \quad \frac{B-K}{2} = \frac{K-V_y}{1} + \frac{K-L}{1} \rightarrow B = 2.5K - V_y - L$$

$$(3) \quad \frac{K-V_y}{1} = \frac{V_y}{2} \qquad K = 1.5V_y$$

$$(3) \quad \frac{K-L}{1} = \frac{L}{2} + \frac{L}{2} \qquad K = 2L$$

$$B = 2.5K - V_y - L = 2.5(1.5V_y) - V_y - \frac{K}{2}$$

$$\frac{B}{2} = 3.75V_y - V_y - \frac{1.5V_y}{2}$$

$$\frac{B}{2} = 2V_y \qquad \boxed{V_y = \frac{B}{4}} \quad (4)$$

and V_3

(2) $\frac{A-L}{2} = \frac{L}{2} + \frac{L-K}{1}$ $\qquad \frac{A}{2} = 2L - K$ $\rightarrow$ $\frac{A}{2} = 2[2.5K - V_z] - K$

$\downarrow$

(3) $\frac{L-K}{1} = \frac{K}{2} + \frac{K-V_z}{1}$ $\qquad L = 2.5K - V_z$ $\qquad \frac{A}{2} = 4K - 2V_z$

$\downarrow$

(3) $\frac{K-V_z}{1} = \frac{V_z}{2}$ $\qquad K = 1.5V_z \longrightarrow \frac{A}{2} = 4(1.5V_z) - 2V_z$

$\downarrow$

$\boxed{V_z = \frac{A}{8}}$ (4) $\longleftarrow \frac{A}{2} = 6V_z - 2V_z = 4V_z$

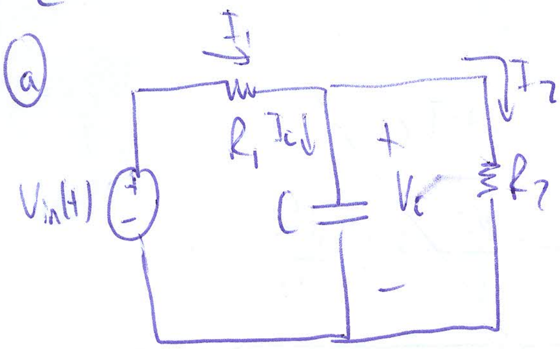
Due to super-position

$D = V_x + V_y + V_z = \frac{C}{2} + \frac{B}{4} + \frac{A}{8}$

(1)

Q-2

(a)



2

$I_1 = I_2 + I_c$

3 $\quad \dfrac{V_{in} - V_c}{R_1} = C\dfrac{dV_c}{dt} + \dfrac{V_c}{R_2}$ $\quad$ 2 $\quad$ 3

5 $\quad \boxed{\dfrac{dV_c}{dt} + \dfrac{1}{C}\left[\dfrac{1}{R_1} + \dfrac{1}{R_2}\right]V_c = \dfrac{1}{CR_1} V_{in}}$

(b) $\dfrac{dx}{dt} + 5x = e^{-5t} + te^{-5t}$

$x_p(t) = (K_1 t + K_2 t^2)e^{-5t}$

$\dfrac{d}{dt} x_p(t) + 5x_p(t) = e^{-5t} + te^{-5t}$

$\dfrac{d}{dt}\left[\left[K_1 t + K_2 t^2\right]e^{-5t}\right] + 5\left(\left[K_1 t + K_2 t^2\right]e^{-5t}\right] = e^{-5t} + te^{-5t}$

8

$(K_1 + 2K_2 t)e^{-5t} + (-5)e^{-5t}\left[K_1 t + K_2 t^2\right] + 5K_1 te^{-5t} + 5K_2 t^2 e^{-5t} = e^{-5t} + te^{-5t}$

$K_1 e^{-5t} + 2K_2 te^{-5t} - 5K_1 te^{-5t} - 5K_2 t^2 e^{-5t} + 5K_1 te^{-5t} + 5K_2 t^2 e^{-5t} = e^{-5t} + te^{-5t}$

$K_1 e^{-5t} + (2K_2 - 5K_1 + 5K_1)te^{-5t} = e^{-5t} + te^{-5t}$ $\quad$ 1 $\quad K_1 = 1$

1 $\quad 2K_2 = 1 \quad K_2 = \dfrac{1}{2}$

$x_p(t) = \left(t + \dfrac{t^2}{2}\right)e^{-5t}$

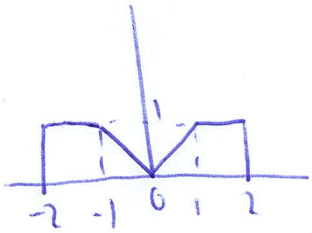
$x_n(t) = Ke^{-5t}$

3 $\quad x(t) = x_n(t) + x_p(t) = \left(t + \dfrac{t^2}{2}\right)e^{-5t} + Ke^{-5t}$

2 $\quad x(0) = K = 2 \longrightarrow x(t) = \left(t - \dfrac{t^2}{2}\right)e^{-5t} + 2e^{-5t}$

Q-3

(8) (a) $y(t) = u(t+2) - r(t+1) + 2r(t) - r(t-1) - u(t-2)$



(12) (b) $I_c(t) = \begin{cases} 2\delta(t) & , 0^- < t < 0^+ \\ \sin(\pi t) & , 0^+ < t < 2 \\ 0 & , \text{otherwise} \end{cases}$

result

$$V_c(t) = \begin{cases} 2 & , 0^- < t < 0^+ \\ 2 + \frac{1}{\pi} - \frac{\cos(\pi t)}{\pi} & , 0^+ < t < 2 \\ 2 & , t > 2 \end{cases}$$

$0^- < t < 0^+$

$$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} I_c(t')\,dt' = 0 + \frac{1}{1}\int_{0^-}^{t} 2\delta(t')\,dt' = 2u(t)$$

$V_c(0^+) = 2$ // 4

$0^+ < t < 2$

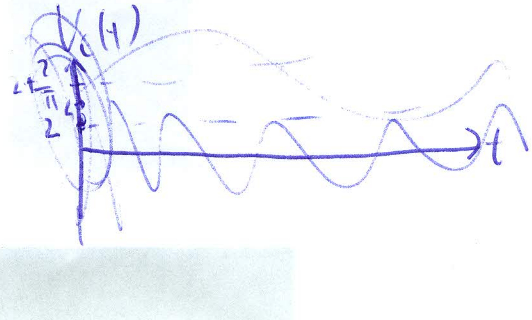
$$V_c(t) = V_c(0^+) + \frac{1}{C}\int_{0^+}^{t} I_c(t')\,dt' = 2 + \frac{1}{1}\int_{0^+}^{t} \sin(\pi t')\,dt'$$

$$= 2 + \frac{1}{\pi}\left(-\cos(\pi t')\right)\Big|_{0^+}^{t} = 2 + \frac{1}{\pi}\left[-\left[\cos(\pi t) - \cos(0\pi)\right]\right]$$ 8

$$= 2 + \frac{1}{\pi}\left[-\left[\cos(\pi t) - 1\right]\right] = 2 - \frac{1}{\pi}\left[\cos(\pi t) - 1\right] = 2 + \frac{1}{\pi} - \frac{\cos(\pi t)}{\pi}$$

$V_c(0^+) = 2$ $\quad V_c(1) = 2 - \frac{1}{\pi}\left[-1-1\right] = 2 + \frac{2}{\pi}$ $\quad V_c(2) = 2 - \frac{1}{\pi}\left[1-1\right] = 2$

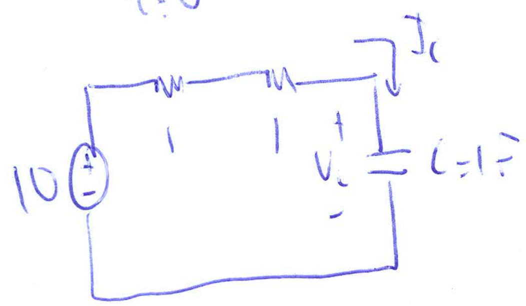
$2 < t \Rightarrow V_c(t) = 2$



(10)

Q-4 -

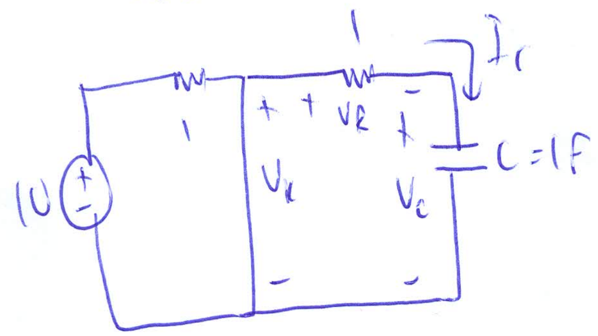
$t = 0^-$



$V_c(0^-) = 5$ Volt (3)

$I_c(0^-) = \frac{10 - V_c(0^-)}{1+1} = \frac{10-5}{2} = \frac{5}{2}$ A

$t = 0^+$



(3)

$V_c(0^+) = V_c(0^-) = 5$ Volt

due to continuity property

$V_x = V_R + V_C$

$V_x(0^+) = 0 = V_R(0^+) + V_C(0^+)$

$0 = 1 \times I_c(0^+) + V_c(0^+)$ (4)

$0 = I_c(0^+) + 5 \quad I_c(0^+) = -5$ //