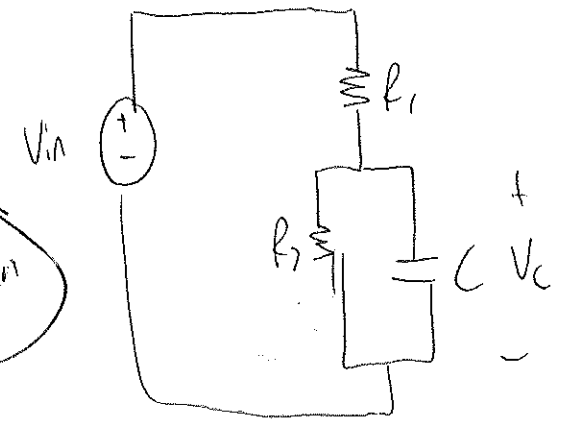
MECE 733 midterm 27-11-2018

Q1



10 puan

$$\frac{V_{in}-V_c}{R_1} = \frac{V_c}{R_2} + C\frac{dV_c}{dt}$$

$$\frac{V_{in}}{R_1} = V_c\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + C\frac{dV_c}{dt}$$

10 puan

$$\frac{dV_c}{dt} + \frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_c = \frac{1}{R_1 C}V_c$$

$$\frac{dV_c}{dt} + 100V_c = 50V_c$$

$$\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = 100$$

2 puan

$$\frac{1}{R_1 C} = 50$$

2 puan

$$\frac{1}{R_1} = 50 \times C$$

$$\frac{1}{R_1} = 50 \times 10^{-6}$$

$$R_1 = \frac{10^6}{50} = \frac{10^3}{5} = 200\Omega$$

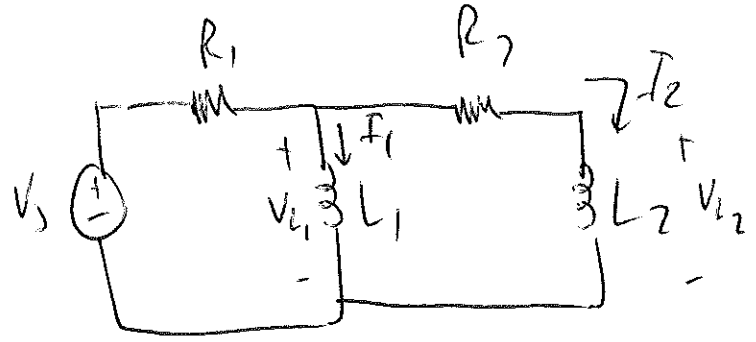
3 puan

$$\frac{1}{R_1}+\frac{1}{R_2} = 100\times 10^{-6} \qquad \frac{1}{R_1}+\frac{1}{R_2} = 10^{-4} \Rightarrow \frac{1}{200}+\frac{1}{R_2} = 10^{-4} \qquad \frac{1}{200}+\frac{1}{R_2} = \frac{1}{10000}$$

$$R_2 = 200\Omega$$

3 puan

(Q2)



$$\frac{V_s - V_{L_1}}{R_1} = I_1 + I_2$$

$L_1 = 1\ H \quad R_1 = 1\ Ohm$

$$\frac{V_s - L_1 \frac{dI_{L_1}}{dt}}{R_1} = I_1 + I_2$$

$$\frac{dI_1}{dt} = V_s - I_1 - I_2 \quad *$$

7 puan

$$V_s = R_1 (I_1 + I_2) + R_2 I_2 + L_2 \frac{dI_2}{dt}$$

$$V_s = (I_1 + I_2) + R_2 I_2 + L_2 \frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \frac{V_s}{L_2} - \frac{1}{L_2} I_1 - \frac{(1+R_2)}{L_2} I_2 \quad **$$

7 puan

$$\frac{d^2 I_1}{dt^2} = \frac{dV_s}{dt} - \frac{dI_1}{dt} - \frac{dI_2}{dt}$$

$$\frac{d^2 I_1}{dt^2} + \frac{dI_1}{dt} = \frac{dV_s}{dt} - \left[\frac{V_s}{L_2} - \frac{1}{L_2} I_1 - \frac{(1+R_2)}{L_2} I_2 \right]$$

$$\frac{d^2 I_1}{dt^2} + \frac{dI_1}{dt} + \frac{1}{L_2} I_1 = \frac{dV_s}{dt} - \frac{V_s}{L_2} + \left(\frac{1+R_2}{L_2} I_2\right)$$

From * → $\boxed{I_2 = V_s - I_1 - \frac{dI_1}{dt}}$

$$\frac{d^2 I_1}{dt^2} + \frac{dI_1}{dt} + \frac{1}{L_2} I_1 = \frac{dV_s}{dt} - \frac{V_s}{L_2} + \left[\frac{1+R_2}{L_2}\left(V_s - I_1 - \frac{dI_1}{dt}\right)\right]$$

$$\frac{d^2 I_1}{dt^2} + \left[1 + \frac{1+R_2}{L_2}\right]\frac{dI_1}{dt} + \left[\frac{1}{L_2} + \frac{1+R_2}{L_2}\right] I_1 = \frac{dV_s}{dt} + \left[\frac{1+R_2}{L_2} - \frac{1}{L_2}\right] V_s$$

7 ρυθμ

char_equation

$$s^2 + \left[1 + \frac{1+R_2}{L_2}\right] s + \left[\frac{1}{L_2} + \frac{1+R_2}{L_2}\right] = 0 \quad = s^2 + 5s + 6$$

5 ρυθμ

$$5 = 1 + \frac{1+R_2}{L_2} \qquad 6 = \frac{1}{L_2} + \frac{1+R_2}{L_2}$$

$$\frac{1+R_2}{L_2} = 4 \qquad 6 = \frac{1}{L_2} + 4 \qquad 2 = \frac{1}{L_2} \qquad L_2 = 0{,}5 \text{ Henry}$$

2 ρυθμ

$$\frac{1+R_2}{0{,}5} = 4 \qquad 1+R_2 = 2 \qquad R_2 = 1\,\Omega$$

2 ρυθμ

(Q3) (a) s^2+5s+6 $s_1=-3$ $s_2=-2$ both of them are negative and real (overdamped system) (2 points)

(b) $\ddot{x}+5\dot{x}+6x=0$ (1 point) $x(0)=1$ $x'(0)=-2$

↓ input is zero

no particular solution since input is zero ($x_p(t)=0$)

$x(t)=x_h(t)+x_p(t)$

↑ h_0

(2 points)

$x_p(t)=0$ $x_h(t)=A_1e^{-3t}+A_2e^{-2t}$

$x(t)=A_1e^{-3t}+A_2e^{-2t}$

$x(0)=A_1+A_2=1$ (2 points)

$\frac{dx}{dt}=-3A_1e^{-3t}-2A_2e^{-2t}$ $\left.\frac{dx}{dt}\right|_{t=0}=-3A_1-2A_2=-2$ (2 points)

$A_1+A_2=1$ $-3A_1-2A_2=-2$

$2A_1+2A_2=2$

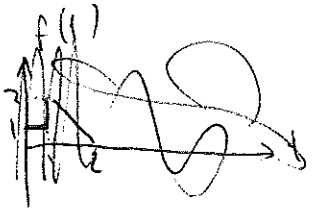
$-3A_1-2A_2=-2$

$-A_1=0$ $A_1=0$ $A_2=1$

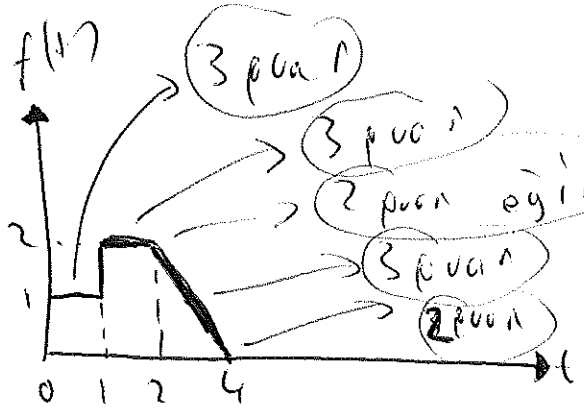
(1 point)

$x(t)=0e^{-3t}+1e^{-2t}=e^{-2t}$

(c) $f(t)=u(t)+u(t-1)-(t-2)-u(t-4)$

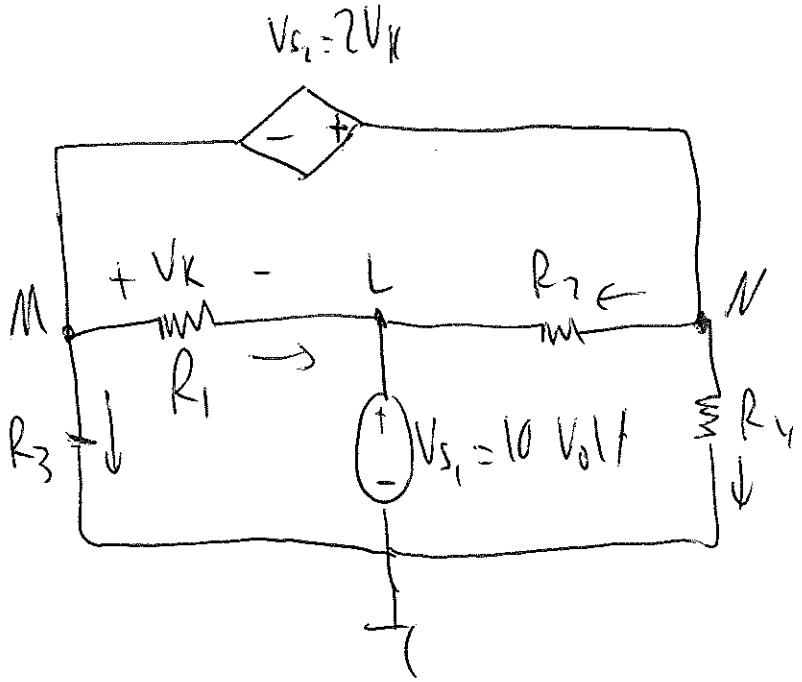


(Q4) $f(t) = u(t)\,tu(t-1) - r(t-2) + r(t-4)$



doğru cevap 2 puan

(Q5)



$R_1 = R_2 = R_3 = 1$

$R_4 = 0.5\ \Omega$

$L = 10$ Volt (1 point)

(10 point) $\frac{M-L}{R_1} + \frac{M}{R_3} + \frac{N-10}{R_2} + \frac{N}{R_4} = 0 \Rightarrow 2M - 10 + 3N - 10 = 0$

$2M + 3N = 20$ (1 point)

$M - 10 = V_K$ (1 point)

$2M - 20 = 2V_K = N - M \Rightarrow N = 3M - 20$ (5 point)

$2M + 3(3M - 20) = 20$

$11M - 60 = 20 \longrightarrow$

$M = \frac{80}{11}$ (2 point)

$N = 3 \times \frac{80}{11} - 20 = \frac{20}{11}$