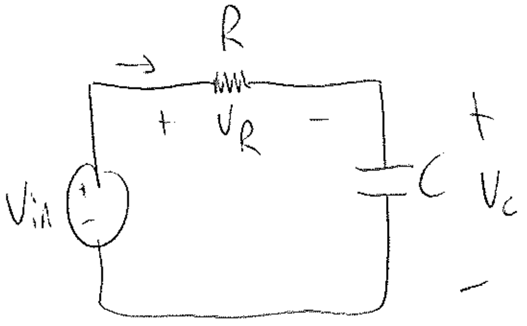
MECE 233 Final

Q1

15 puan



$C\frac{dV_c}{dt}R + V_c = V_{in}$

$V_{in} = V_R + V_c$ (3)

$V_{in} = RI_R + V_c$ (3)

$V_{in} = RI_C + V_c$ (3)

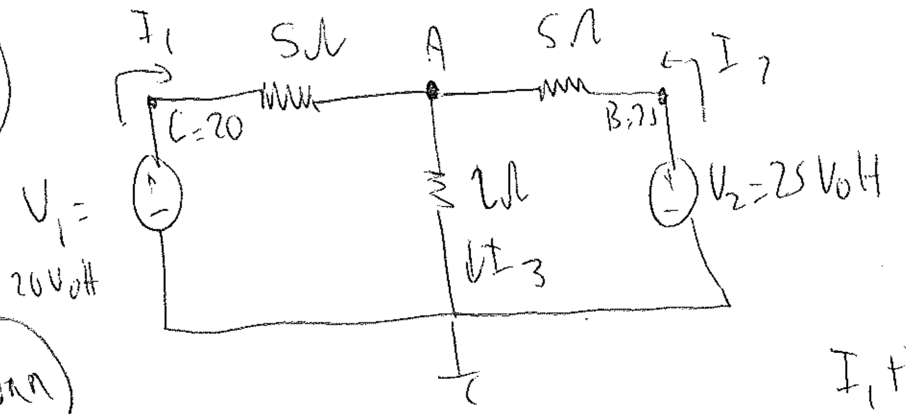
$V_{in} = RC\frac{dV_c}{dt} + V_c$ (6)

$i_c = C\frac{dV_c}{dt}$

$I_c = I_R =$

Q2

15 puan



$C = 20 = V_1$

2 puan

$B = 25 = V_2$

2 puan

$I_1 + I_2 = I_3$

$\frac{V_1 - A}{5} + \frac{V_2 - A}{5} = \frac{A}{2}$

$\frac{20 - A}{5} + \frac{25 - A}{5} = \frac{A}{2}$ 5 puan

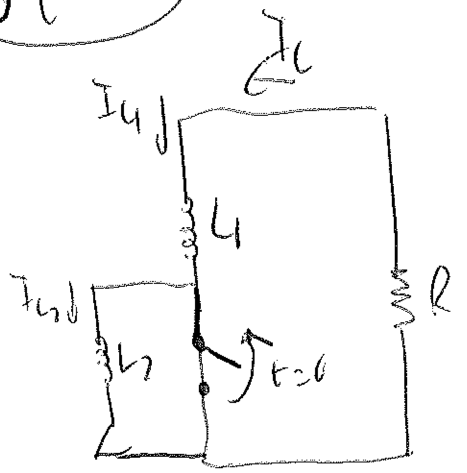
$2(20 - A + 25 - A) = 5A$

$90 = 9A$ $A = 10$ Volt

6 puan

25 puan

Q3



$I_{L_1}(0) = 1$ Ampere

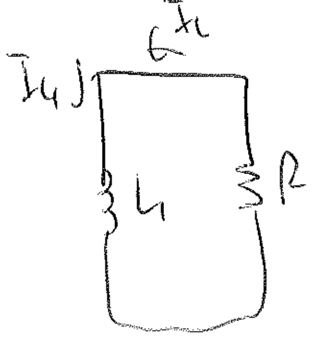
$I_{L_2}(0) = \frac{5}{2}$ Ampere

$L_1 = 1\ H$

$L_2 = 2$ Henry

Gidiş 10 puan

$0 < t < \ln 2$



$I_{L_1}(0) = 2$ Ampere

$L\frac{dI_{L_1}}{dt} = V_R = R(-I_{L_1})$ $\quad L\frac{dI_{L_1}}{dt} + R I_{L_1} = 0$

$$\frac{dI_{L_1}}{dt} + \frac{R}{L} I_{L_1} = 0 \Rightarrow I_{L_1} = k e^{-t}$$

2 puan

$R = 1\Omega \quad L = 1H$

$I_{L_1}(0) = 2e^{-t}$ (2 puan)

$I_1(\ln 2) = 2e^{-\ln 2}$

$= 2e^{\ln 2^{-1}}$

$= 2e^{\ln \frac{1}{2}} = 2 \times \ldots$

$= 1$ Ampere

1 puan

$I_L = I_{L_1}$

at $t = \ln 2$

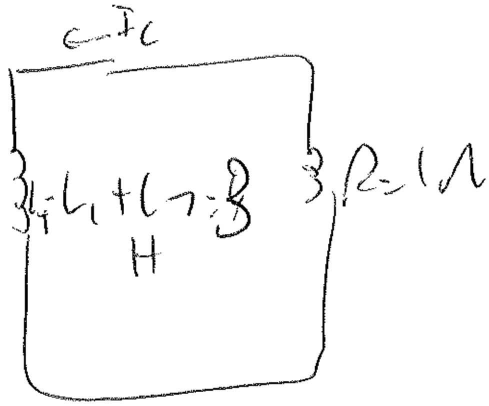
$$L_1 I_{L_1}(\ln 2^-) + L_2 I_{L_2}(\ln 2^-) = (L_1 + L_2) I_L(\ln 2^+)$$

$$1 + 1 \quad + 2 \times \frac{5}{2} = (3)\ I_L(\ln 2^+)$$

$$I_L(\ln 2^+) = \frac{6}{3} = 2 \text{ Ampere}$$

2 puan

t) ln 2



$$\frac{dI_L}{dt} + \frac{R}{L_T} I_L = 0 \qquad \frac{dI_L}{dt} + \frac{1}{3} I_L = 0$$

(3 puan)

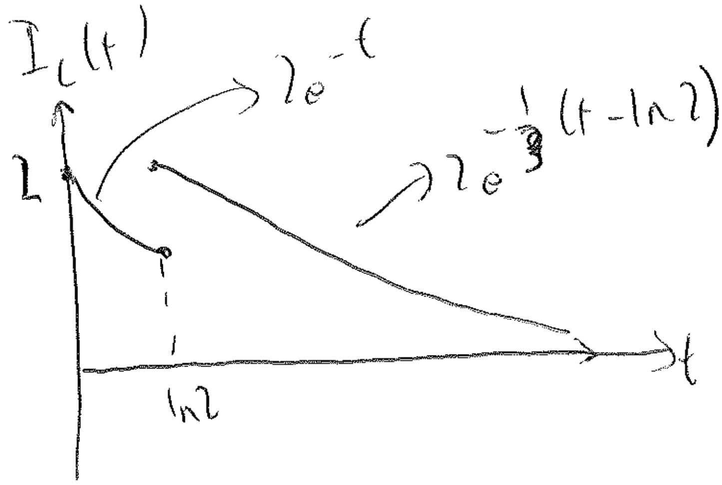
$$I_L(t) = M e^{-\frac{(t)}{3}} = N e^{-\frac{1}{3}(t - \ln 2)}$$

$$2 = I_L(\ln 2^+) = N e^{-\frac{(\ln 2 - \ln 2)}{3}} = N \qquad N = 2$$

(2 puan)

$$I_L(t) = 2 e^{-\frac{1}{3}(t - \ln 2)}$$

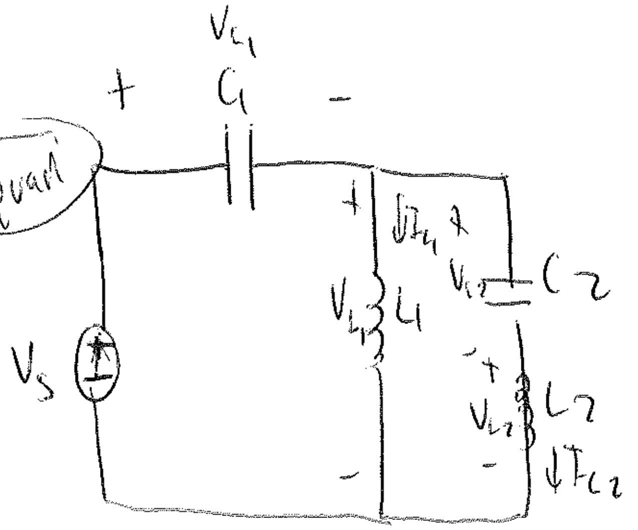
$$I_L(t) = \begin{cases} 2e^{-t} & 0 \leq t < \ln 2 \\ 2e^{-\frac{1}{3}(t - \ln 2)} & \ln 2 \leq t \end{cases}$$



(3 puan)

Q4

31 puan



$V_s = V_{C_1} + V_{C_2} + V_{L_2}$

$V_s = V_{C_1} + V_{C_2} + L_2 \frac{dI_{L_2}}{dt}$

$$\boxed{\frac{dI_{L_1}}{dt} = \frac{V_s}{L_1} - \frac{V_{C_1}}{L_1}}$$

$$\boxed{\frac{dI_{L_2}}{dt} = \frac{V_s}{L_2} - \frac{V_{C_1}}{L_2} - \frac{V_{C_2}}{L_2}}$$

15 puan

State equations

$V_s = V_{C_1} + V_{L_1}$

$V_s = V_{C_1} + L_1 \frac{dI_{L_1}}{dt}$

$V_{L_1} = V_{C_2} + V_{L_2}$

$L_1 \frac{dI_{L_1}}{dt} = V_{C_2} + L_2 \frac{dI_2}{dt}$

$I_{L_2} = C_2 \frac{dV_{C_2}}{dt}$

$$\boxed{\frac{dV_{C_2}}{dt} = \frac{1}{C_2} I_{L_2}}$$

$I_{C_1} = I_{L_1} + I_{L_2}$

$C_1 \frac{dV_{C_1}}{dt} = I_{L_1} + I_{L_2} \Rightarrow$ $\boxed{\frac{dV_{C_1}}{dt} = \frac{1}{C_1} I_{L_1} + \frac{1}{C_1} I_{L_2}}$

$$\begin{bmatrix} \frac{dV_{C_1}}{dt} \\ \frac{dV_{C_2}}{dt} \\ \frac{dI_{L_1}}{dt} \\ \frac{dI_{L_2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & 0 & 0 & \frac{1}{C_2} \\ -\frac{1}{L_1} & 0 & 0 & 0 \\ -\frac{1}{L_2} & -\frac{1}{L_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{C_1} \\ V_{C_2} \\ I_{L_1} \\ I_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} V_s$$

16 puan

$C_1 = 2 \quad L_1 = 2$

$C_2 = 1 \quad L_2 = 1$

$$\Rightarrow \begin{bmatrix} \frac{dV_{C_1}}{dt} \\ \frac{dV_{C_2}}{dt} \\ \frac{dI_{L_1}}{dt} \\ \frac{dI_{L_2}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \\ -\frac{1}{2} & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_{C_1} \\ V_{C_2} \\ I_{L_1} \\ I_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix}$$

Q5 $A(t) = 5\cos(5t + 90^\circ) \xrightarrow{\text{phasor}} 5e^{j90^\circ} = 5\left[\cos 90^\circ + j\sin 90^\circ\right]$

$= 5j$

4 puan

Q6 $\dfrac{d^3x}{dt^3} + \dfrac{d^2x}{dt^2} + \dfrac{3dx}{dt} + 3x = u$

6 puan

(a) char-equation $s^3 + s^2 + 3s + 3$ — 2 point

(b) $s^2(s+1) + 3(s+1) = 0 \qquad (s^2+3)(s+1) = 0$

$s_1 = +\sqrt{3}j \quad s_2 = -\sqrt{3}j \quad s_3 = -1$ → natural frequencies — 2 point

(c) $x_h = K_1 e^{\sqrt{3}j} + K_2 e^{-\sqrt{3}j} + K_3 e^{t}$

$x_h = M_1 \cos(\sqrt{3}t) + M_2 \sin(\sqrt{3}t) + M_3 e^{t}$ — 2 point

(d) $x_p = A\cos(3t) + B\sin(3t)$

(e) $x_p = At\cos(\sqrt{3}t) + Bt\sin(\sqrt{3}t)$

Q7 10 puan