$$i_{Ln}(t) = B_1 e^{s_1 t} + B_2 e^{s_2 t}$$
$$i_{Lp} = I_s$$
$$\left.\right\} \quad i_L = B_1 e^{s_1 t} + B_2 e^{s_2 t} + I_s$$

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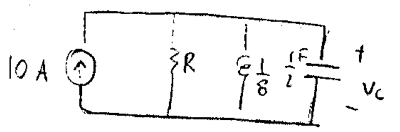
$$\left.\begin{aligned} B_1 + B_2 + I_s &= I_0 \\ s_1 B_1 + s_2 B_2 &= \frac{V_0}{L} \end{aligned}\right\} \text{ solve for } B_1 \text{ \& } B_2$$

$i_L(t) \to I_s$ as $t \to \infty$

steady state of $i_L(t)$ is I_s.

Exercise:

$*$



$i_L(0) = -4$
$v_c(0) = 5\,V$

Find $v_c(t)$ and $i_L(t)$ for the same cases as in the previous example.

answer:

i) $R = \frac{1}{5}\Omega$ $\quad v_c(t) = 3e^{-2t} + 2e^{-8t}$,
$$i_L(t) = -12e^{-2t} - 2e^{-8t} + 10$$

ii) $R = \frac{1}{4}\Omega$, $v_c(t) = (5 + 8t)\, e^{-4t}$
$$i_L(t) = (-14 - 16t)\, e^{-4t} + 10$$

iii) $R = \frac{1}{3}\Omega$, $\quad v_c(t) = e^{-3t}\, 2\sqrt{\frac{86}{7}} \cos\left(\sqrt{7}\,t - \tan^{-1}\frac{13}{5\sqrt{7}}\right)$
$$i_L(t) = -4\sqrt{\frac{86}{7}}\, e^{-3t} \cos\left(\sqrt{7}\,t - \tan^{-1}\left(\frac{1}{7\sqrt{7}}\right)\right) + 10$$

iv) $R \to \infty$ $\quad v_c(t) = \sqrt{74} \cos(4t - 54.46^\circ)$
$$i_L(t) = -2\sqrt{74} \cos\left(4t + \tan^{-1}\frac{5}{7}\right) + 10$$

Ex: $I_s = 10$ ECE 233-281 Ulaş Beldek

$I_L(0) = -4$ A

$V_C(0) = 5$ V

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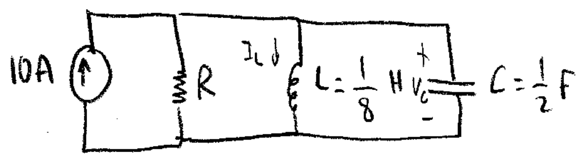

Find V_C and I_L

(i) $R = \frac{1}{5}\,\Omega$

(over damped case)

$$\frac{d^2V_c}{dt^2} + 2\alpha \frac{dV_c}{dt} + \omega_0^2 V_c = \frac{1}{C}\frac{dI_s}{dt} \qquad 2\alpha = \frac{1}{RC} \quad \omega_0^2 = \frac{1}{LC}$$

$V_c(0) = 5 \qquad \frac{dV_c}{dt}(0) = \frac{1}{C}\frac{I_s(0)}{1} - \frac{1}{C}\frac{I_L(0)}{1} - \frac{1}{RC}\frac{V_c(0)}{1}$

$$\frac{d^2I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + \omega_0^2 I_L = \omega_0^2 I_s \qquad I_L(0) = -4$$

$$\frac{dI_L}{dt}(0) = \frac{V_c(0)}{L}$$

$S_{1,2} = -\alpha \mp \sqrt{\alpha^2 - \omega_0^2} \qquad \alpha = 5 \quad \omega_0 = 4$

$S_{1,2} = -2, -8$

For $V_c(t) = V_{cp} + V_{ch}$

(a) For particular solution V_{cp}

$$\frac{d^2V_c}{dt^2} + 10\frac{dV_c}{dt} + 16V_c = 2\underbrace{\frac{dI_s}{dt}}_{0} = 0 \quad \left(\begin{array}{l}\text{since homogeneous}\\ \text{diff equation}\end{array}\right) \Rightarrow V_{cp} = 0$$

(for all cases)

$V_c(0) = 5$

$$\frac{dV_c}{dt}(0) = \frac{10}{\frac{1}{2}} - \frac{(-4)}{\frac{1}{2}} - \frac{5}{\frac{1}{2}\times\frac{1}{5}} = 20 + 8 - 50 = -22$$

(b) $V_{ch} = A_1 e^{-2t} + A_2 e^{-8t}$

(c) $V_c(t) = 0 + A_1 e^{-2t} + A_2 e^{-8t}$

$$\left.\begin{array}{l} V_c(0) = 5 = A_1 + A_2 \\ \frac{dV_c(0)}{dt} = -22 = -2A_1 - 8A_2 \end{array}\right\} \begin{array}{l} A_1 = 3 \\ A_2 = 2 \end{array}$$

$$\boxed{V_c(t) = 3e^{-2t} + 2e^{-8t}}$$

(2)

For $I_L(t) = I_{Lp} + I_{Lh}$

$$\frac{d^2 I_L}{dt^2} + 10\frac{dI_L}{dt} + 16 I_L = 16 * \underset{10}{\underset{\sim}{I_s}} \qquad I_L(0) = -4A \qquad \frac{dI_L}{dt}(0) = \frac{V_c(0)}{L} = \frac{5}{\frac{1}{8}} = 40$$

(a) For $I_{Lp} = K$ (constant)

$$\underbrace{\frac{d^2}{dt^2}K}_{0} + 10\underbrace{\frac{d}{dt}K}_{0} + 16K = 16 * 10$$

$$0 + 16K = 160 \rightarrow K = 10 \qquad I_{Lp} = 10 \quad \left(\begin{matrix}\text{for} \\ \text{all} \\ \text{cases}\end{matrix}\right)$$

(b) $I_L(t) = I_{Lp} + I_{Lh} = 10 + I_{Lh} = 10 + B_1 e^{-2t} + B_2 e^{-8t}$

(c) $I_L(0) = 10 + B_1 + B_2 = -4 \qquad \frac{dI_L}{dt}(0) = -2B_1 + -8B_2 = 40$

$B_1 = -12 \quad B_2 = -2$

$$\boxed{I_L(t) = -12e^{-2t} - 2e^{-8t} + 10}$$

(ii) $R = \frac{1}{4} \qquad \alpha = 4 \quad w_0 = 4 \quad \alpha = w_0$ (critically damped case)

$s_{1,2} = -4 = -\alpha \qquad V_c(0) = 5 \qquad \frac{dV_c}{dt}(0) = -\frac{V_c(0)}{RC} - \frac{I_L(0)}{C} + \frac{I_s(0)}{C} = -40+8+20 = -12$

For $V_c(t) = V_{ch} + V_{cp}$

(a) $V_{cp} = 0$ (since it is the same as in part i-a

(b) $V_{ch} = (K_1 + K_2 t)e^{-4t}$

(c) $V_c(t) = 0 + (K_1 + K_2 t)e^{-4t} \qquad V_c(0) = 5 = K_1$

$\qquad \frac{dV_c}{dt}(0) = K_2 - 4K_1 = -12 \Big\} \quad K_2 = 8$

$$\boxed{V_c(t) = (5 + 8t)\, e^{-4t}}$$

For $I_L(t) = I_{Lp} + I_{Lh}$

(a) $I_{Lp} = 10$ (as in i-a)

(b) $I_{Lh} = (M_1 + M_2 t)e^{-4t}$

(c) $I_L = 10 + (M_1 + M_2 t)e^{-4t}$

$I_L(0) = -4 = 10 + M_1$ $\quad \frac{dI_L}{dt}(0) = -4M_1 + M_2 = \frac{V_c(0)}{L} = \frac{5}{\frac{1}{8}} = 40$

$M_1 = -14 \quad M_2 = -16$

$$\boxed{I_L(t) = (-14 - 16t)e^{-4t} + 10}$$

(iii) $R = \frac{1}{3}\Omega \quad \alpha = 3 \quad \omega_0 = 4 \quad s_{1,2} = -3 \mp j\sqrt{4^2 - 3^2} = -3 \mp j\sqrt{7}$

(underdamped case)

For $V_c(t) = V_{cp}(t) + V_{ch}(t)$

(a) $V_{cp}(t) = 0$ (as in part i-a)

(b) $V_{ch}(t) = e^{-3t}\left(A_1 \cos(\sqrt{7}t) + A_2 \sin(\sqrt{7}t)\right)$

$V_c(0) = 5 = A_1$ $\quad \frac{dV_c}{dt}(0) = \frac{I_s(0)}{C} - \frac{I_L(0)}{C} - \frac{V_c(0)}{RC} = \frac{10}{\frac{1}{2}} - \frac{(-4)}{\frac{1}{2}} - \frac{5}{(\frac{1}{3})(\frac{1}{2})}$

$\frac{dV_c}{dt}(0) = 28 - 30 = -2$

$= -3A_1 + \sqrt{7}A_2 = -2$

$A_2 = \frac{13}{\sqrt{7}}$ $\quad \boxed{V_c(t) = e^{-3t}\left[5\cos(\sqrt{7}t) + \frac{13}{\sqrt{7}}\sin(\sqrt{7}t)\right]}$

(4)

or

$$V_c(t) = \sqrt{5^2 + \left(\frac{13}{\sqrt{7}}\right)^2} \; e^{-3t} \cos\left(\sqrt{7}t - \arctan \frac{13}{\sqrt{7} * 5}\right)$$

$$= \sqrt{\frac{25*7+169}{7}} \; e^{-3t} \cos\left(\sqrt{7}t - \arctan \frac{13}{5*\sqrt{7}}\right)$$

$$\boxed{V_c(t) = 2\sqrt{\frac{86}{7}} \; e^{-3t} \cos\left(\sqrt{7}t - \arctan \frac{13}{5*\sqrt{7}}\right)}$$

For $I_L(t) = I_{Lp} + I_{Ln}$

(a) $I_{Lp} = 10$ (as in part i-a)

(b) $I_{Ln} = e^{-3t}\left[B_1 \cos(\sqrt{7}t) + B_2 \sin(\sqrt{7}t)\right]$

(c) $I_L = 10 + e^{-3t}\left[B_1 \cos(\sqrt{7}t) + B_2 \sin(\sqrt{7}t)\right]$

$I_L(0) = -4 = 10 + B_1 \qquad B_1 = -14$

$\frac{dI_L}{dt}(0) = \frac{V_c(0)}{L} = \frac{5}{\frac{1}{8}} = 40 = -3B_1 + \sqrt{7}B_2 \qquad B_2 = \frac{-2}{\sqrt{7}}$

$$I_L(t) = \boxed{10 + e^{-3t}\left[-14 \cos(\sqrt{7}t) - \frac{2}{\sqrt{7}} \sin(\sqrt{7}t)\right]}$$

$$= 10 - e^{-3t}\left[14 \cos(\sqrt{7}t) + \frac{2}{\sqrt{7}} \sin(\sqrt{7}t)\right]$$

$$= 10 - e^{-3t} \sqrt{14^2 + \left(\frac{2}{\sqrt{7}}\right)^2}\left[\cos\left(\sqrt{7}t - \arctan \frac{\frac{2}{\sqrt{7}}}{14}\right)\right]$$

$$\boxed{I_L(t) = 10 - e^{-3t} \, 4\sqrt{\frac{86}{7}} \cos\left(\sqrt{7}t - \arctan \frac{1}{7\sqrt{7}}\right)}$$

(iv) if $R = \infty$ $\alpha = 0$ $\omega_0 = 4$ $s_{1,2} = -\alpha \mp j\sqrt{\omega_0^2 - \alpha^2}$

$= \mp j4$

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(purely sinusoidal case)

For $V_c(t) = V_{ch}(t) + V_{cp}(t)$

(a) $V_{cp} = 0$ (as in part i-a)

(b) $V_{ch} = A_1 \cos(4t) + A_2 \sin(4t)$

(c) $V_c(t) = 0 + A_1 \cos(4t) + A_2 \sin(4t)$

$V_c(0) = 5 = A_1$ $\quad \frac{dV_c}{dt}(0) = \frac{I_s(0)}{C} - \frac{I_L(0)}{C} - \frac{V_c(0)}{RC}$

$= \frac{10}{\frac{1}{2}} - \frac{(-4)}{\frac{1}{2}} - 0 = 28$

$\frac{dV_c}{dt} = 28 = 4A_2 \quad A_2 = 7$

$V_c(t) = \boxed{5\cos(4t) + 7\sin(4t)} = \sqrt{5^2+7^2}\cos\left(4t - \arctan\frac{7}{5}\right)$

$\boxed{V_c(t) = \sqrt{74}\cos\left(4t - \arctan\frac{7}{5}\right)}$

For $I_L(t) = I_{Lh}(t) + I_{Lp}(t)$

(a) $I_{Lp} = 10$; (b) $I_{Lh} = B_1 \cos(4t) + B_2 \sin(4t)$

(c) $I_L(t) = 10 + B_1 \cos(4t) + B_2 \sin(4t)$

$I_L(0) = -4 = 10 + B_1 \quad B_1 = -14$

$\frac{dI_L}{dt}(0) = \frac{V_c(0)}{L} = 40 = 4B_2 \quad B_2 = 10$

$I_L(t) = \boxed{10 - (14\cos(4t) - 10\sin(4t))} = \boxed{10 - 2\sqrt{74}\cos\left(4t - \arctan\frac{-5}{7}\right)}$