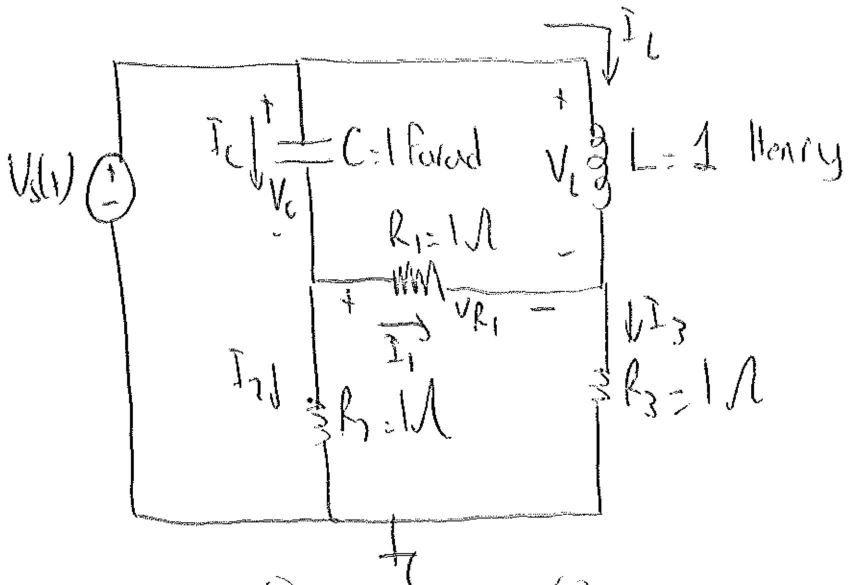
Q-1



$$V_L = L\frac{dI_L}{dt} = \frac{dI_L}{dt}$$ 1

$$C\frac{dV_C}{dt} = I_C = \frac{dV_C}{dt}$$ 1

$$I_C = I_1 + I_2$$ 1

$$C\frac{dV_C}{dt} = \frac{(V_s - V_C) - (V_s - V_L)}{R_1} + \frac{V_s - V_C - 0}{R_2}$$ 2 2

$$1\frac{dV_C}{dt} = \frac{V_L - V_C}{1} + \frac{V_s - V_C}{1}$$

$$\frac{dV_C}{dt} = \frac{L\frac{dI_L}{dt} - V_C}{1} + \frac{V_s - V_C}{1}$$

$$\boxed{\frac{dV_C}{dt} = V_s - 2V_C + \frac{dI_L}{dt}}$$ 5

$$I_L + I_1 = I_3$$ 1

$$I_L + \frac{V_s - V_C - (V_s - V_L)}{1} = \frac{V_s - V_L}{1}$$ 2

$$\rightarrow I_L + \frac{V_L - V_C}{1} = \frac{V_s - V_L}{1}$$

$$\downarrow$$

$$2V_L = V_s - I_L + V_C$$

$$2 \cdot L \cdot \frac{dI_L}{dt} = V_s - I_L + V_C \longrightarrow \boxed{\frac{dI_L}{dt} = \frac{1}{2}V_s - \frac{1}{2}I_L + \frac{1}{2}V_C}$$ * 5

2 $$\boxed{V_C = 2\frac{dI_L}{dt} + I_L - V_s}$$ ***

$$\frac{dV_c}{dt} = V_s - 2V_c + \frac{dI_L}{dt} = V_s - 2V_c + \frac{1}{2}V_s - \frac{1}{2}I_L + \frac{1}{2}V_c$$

$$(3) \quad \boxed{\frac{dV_c}{dt} = \frac{3}{2}V_s - \frac{3}{2}V_c - \frac{1}{2}I_L} \; **$$

Take derivative of * put ** inside

$$\frac{d^2 I_L}{dt^2} = \frac{1}{2}\frac{dV_s}{dt} - \frac{1}{2}\frac{dI_L}{dt} + \frac{1}{2}\frac{dV_c}{dt}$$

$$(3) \quad \frac{d^2 I_L}{dt^2} + \frac{1}{2}\frac{dI_L}{dt} = \frac{1}{2}\frac{dV_s}{dt} + \frac{1}{2}\left[\frac{3}{2}V_s - \frac{3}{2}V_c - \frac{1}{2}I_L\right]$$

(use ***)

$$\frac{d^2 I_L}{dt^2} + \frac{1}{2}\frac{dI_L}{dt} + \frac{1}{4}I_L = \frac{1}{2}\frac{dV_s}{dt} + \frac{3}{4}V_s - \frac{3}{4}V_c$$

$$\frac{d^2 I_L}{dt^2} + \frac{1}{2}\frac{dI_L}{dt} + \frac{1}{4}I_L = \frac{1}{2}\frac{dV_s}{dt} + \frac{3}{4}V_s - \frac{3}{4}\left[2\frac{dI_L}{dt} + I_L - V_s\right]$$

$$\frac{d^2 I_L}{dt^2} + \left(\frac{1}{2}+\frac{3}{2}\right)\frac{dI_L}{dt} + \left[\frac{1}{4}+\frac{3}{4}\right]I_L = \frac{1}{2}\frac{dV_s}{dt} + \left(\frac{3}{4}+\frac{3}{4}\right)V_s$$

$$\boxed{\frac{d^2 I_L}{dt^2} + 2\frac{dI_L}{dt} + 1 I_L = \frac{1}{2}\frac{dV_s}{dt} + \frac{3}{2}V_s} \quad (7)$$

(b) Characteristic equation $\quad$ Natural frequencies

$\hookrightarrow s^2+2s+1=0 \qquad s_{1,2}=-1$

$I_{Lh}(t)=[A_1+A_2t]e^{-t} \longrightarrow I_L$ homogenous solution

Q-2 $\quad s^2+As+B=0 \quad A\geq 0,\ B>0 \quad$ roots $: s_{1,2}=\frac{-A\mp\sqrt{A^2-4B}}{2}$

(a) Overdamped $\Rightarrow$ two real roots, both roots are negative

$A^2-4B>0 \qquad s_1=\frac{-A+\sqrt{A^2-4B}}{2}<0 \quad s_2=\frac{-A-\sqrt{A^2-4B}}{2}<0$

$\rightarrow$ automatically satisfied

(b) Critically-damped $\Rightarrow$ two real roots which are equal to each other

$A^2-4B=0 \qquad s_{1,2}=-\frac{A}{2}<0$ (automatically satisfied)

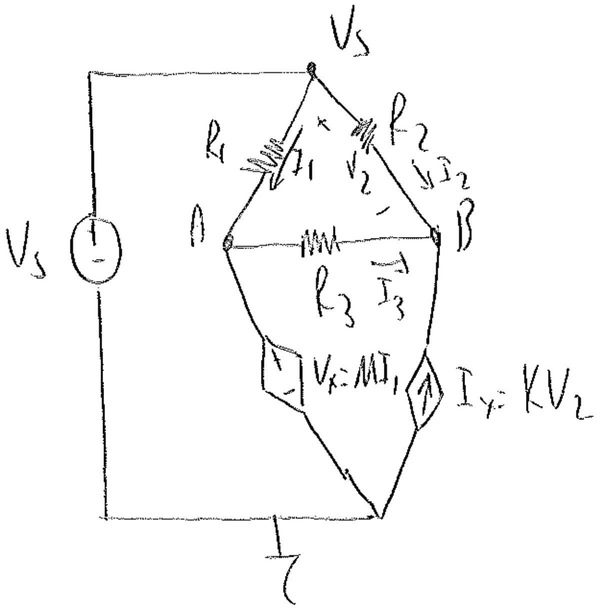
(c) Underdamped case $\Rightarrow$ two complex conjugate root pairs with Real parts negative

$s_{1,2}=-\frac{A}{2}\mp j\sqrt{4B-A^2} \qquad A^2-4B<0$

(d) $A=0 \qquad 4B-A^2>0 \qquad 4B>0 \qquad B>0$

purely oscillatory case.

Q-3-



$$I_1 = \frac{V_s - A}{R_1} \quad 2$$

$$A = V_x = MI_1 = M\frac{V_s - A}{R_1} \quad 2$$

$$A = M\frac{V_s - A}{R_1}$$

$$\boxed{A = \frac{V_s M}{R_1 + M}} \quad 2 \quad **$$

1

$$I_2 + I_3 + I_x = 0 \qquad V_2 = V_s - B \quad 1$$

$$1 \quad \frac{V_s - B}{R_2} + \frac{A - B}{R_3} + KV_2 = 0 \quad 1$$

$$\frac{V_s - B}{R_2} + \frac{A - B}{R_3} + K(V_s - B) = 0 \quad 2$$

$$\frac{V_s}{R_2} + \frac{A}{R_3} + KV_s = B\left[\frac{1}{R_2} + \frac{1}{R_3} + K\right]$$

$$V_s\left[\frac{1}{R_2} + \frac{M}{(R_1+M)R_3} + K\right] = B\left[\frac{1}{R_2} + \frac{1}{R_3} + K\right]$$

$$4 \quad \boxed{B = \frac{V_s\left[\frac{1}{R_2} + K + \frac{M}{(R_1+M)R_3}\right]}{\left[\frac{1}{R_2} + \frac{1}{R_3} + K\right]}} \quad **$$

Q - 4 -

$$V_L(t) = \begin{cases} 1 \text{ Volt}, & 0<t<1 \\ -1 \text{ Volt}, & 1<t<2 \\ \delta(t-2) \text{ Volt}, & t=2 \\ -\delta(t-3) \text{ Volt}, & t=3 \\ 0 \text{ Volt}, & \text{otherwise} \end{cases}$$

$I_L(0) = 0$ Amper

$$I_L(t) = I_L(t_0) + \frac{1}{L}\int_{t_0}^{t} V_L(t')dt'$$

$0<t<1$ $\quad V_L = 1$ Volt

$$I_L(t) = I_L(0) + \frac{1}{1}\int_0^t 1dt' = 0 + t'\Big|_0^t = t$$

2

$I_L(1) = 1$ Amper

$1<t<2$ $\quad V_L = -1$ Volt

$$I_L(t) = I_L(1) + \frac{1}{1}\int_1^t -1dt' = 1 + (-t')\Big|_1^t = 1 + [-(t-1)] = 2-t$$

3

$I_L(1) = 1$ Amper

$I_L(2^-) = 0$ Amper

$t=2$ $\quad V_L = \delta(t-2)$ Volt

$$I_L(t) = I_L(2^-) + \frac{1}{1}\int_{2^-}^{t} \delta(t'-2)dt' = 0 + u(t-2) = u(t-2)$$

$I_L(2^-) = u(2^- - 2) = 0$ Amper

$I_L(2^+) = u(2^+ - 2) = 1$ Amper

2

$2<t<3$ $\quad V_L = 0$ Volt

$$I_L(t) = I_L(2^+) + \frac{1}{1}\int_{2^+}^{t} 0dt' = I_L(2^+) = 1 \text{ Amper}$$

$I_L(3^-) = 1$ Amper

$t=3 \qquad V_L(t) = -\delta(t-3)$ Volt

$$I_L(t) = I_L(3^-) + \frac{1}{1}\int_{3^-}^{t} -\delta(t'-3)\,dt' = 1 - u(t-3)$$

$I_L(3^-) = 1 - u(3^- - 3) = 1$ Amper

(2) $I_L(3^+) = 1 - u(3^+ - 3) = 0$ Amper

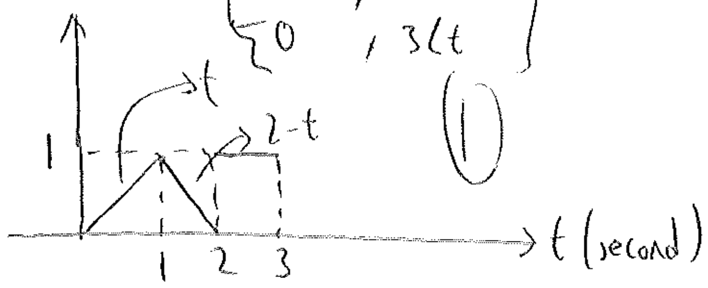
$t>3 \qquad V_L(t) = 0$

$$I_L(t) = I_L(3^+) + \frac{1}{1}\int_{3^+}^{t} 0\,dt' = I_L(3^+) = 0 \text{ Amper}$$

(a)

$$I_L(t) = \begin{cases} t, & 0<t<1 \\ 2-t, & 1<t<2 \\ 1, & 2<t<3 \\ 0, & 3<t \end{cases}$$

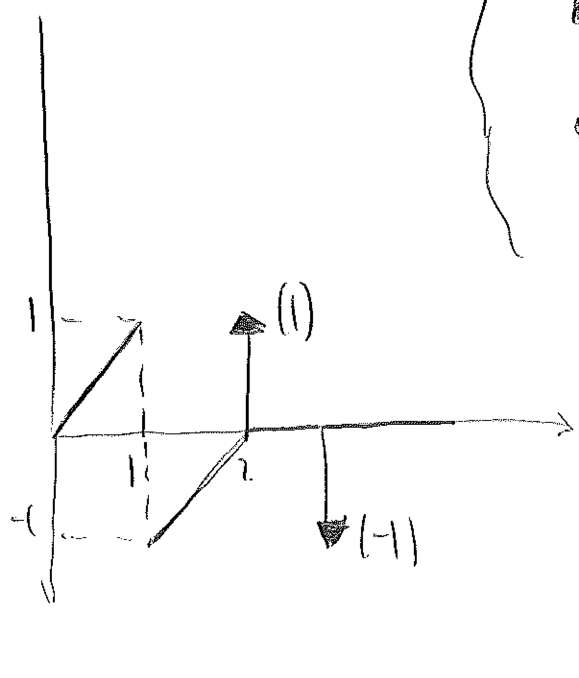
(1)



(b)

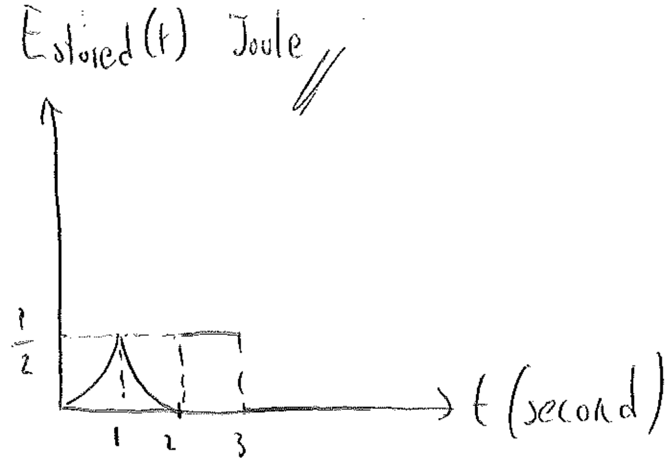
$$P_L(t) = I_L(t)\,V_L(t) = \begin{cases} 1\times t \text{ Watt}, & 0<t<1 \\ -1\times(2-t) \text{ Watt}, & 1<t<2 \\ 1\,\delta(t-2), & t=2 \\ 0 \text{ Watt}, & 2<t<3 \\ -1\,\delta(t-3), & t=3 \\ 0 \text{ Watt} & t>3 \end{cases}$$

(1) (3)

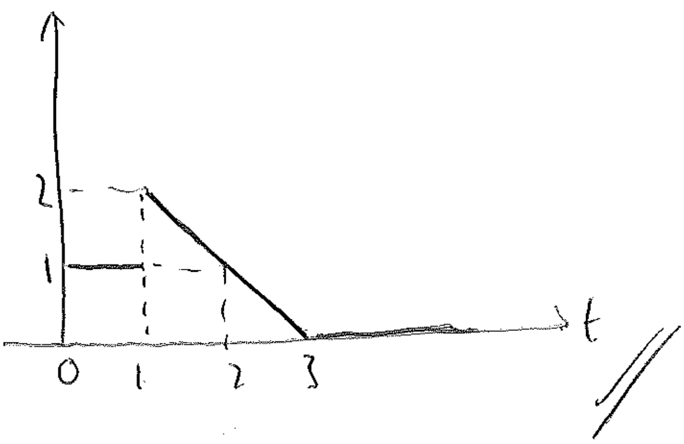


(c) $E_{stored} = \frac{1}{2} L I_L^2(t) - \frac{1}{2} L I_L^2(0) = \frac{1}{2} \times 1 \left[I_L^2(t) - I_L^2(0) \right] = \frac{1}{2} \times 1\, I_L^2(t)$

$$E_{stored} = \begin{cases} \frac{1}{2} t^2, & 0<t<1 \\ \frac{1}{2}(2-t)^2, & 1<t<2 \\ \frac{1}{2} 1^2, & 2<t<3 \\ 0, & t>3 \end{cases} \quad 2$$



Q-5 $k(t) = u(t) + u(t-1) - r(t-1) + r(t-3)$



Q-6 $\quad x_p(t) = te^{-t}\left(K_1 \sin(t) + K_2 \cos(t)\right)$

S $\quad \dfrac{d x_p(t)}{dt} = e^{-t}\left[K_1 \sin(t) + K_2 \cos(t)\right] + te^{-t}\left[K_1 \cos(t) - K_2 \sin(t)\right] - te^{-t}\left[K_1 \sin(t) + K_2 \cos(t)\right]$

$$= e^{-t}\left[K_1 \sin(t) + K_2 \cos(t)\right] + te^{-t}\left[(K_1 - K_2)\cos(t) + (-K_1 - K_2)\sin(t)\right]$$

$$\frac{d^2 x_p(t)}{dt^2} = e^{-t}\left[K_1 \cos(t) - K_2 \sin(t)\right] - e^{-t}\left[K_1 \sin(t) + K_2 \cos(t)\right]$$
$$+ e^{-t}\left[(K_1 - K_2)\cos(t) + (-K_1 - K_2)\sin(t)\right]$$
$$- te^{-t}\left[(K_1 - K_2)\cos(t) + (-K_1 - K_2)\sin(t)\right]$$
$$+ te^{-t}\left[-(K_1 - K_2)\sin(t) + (-K_1 - K_2)\cos(t)\right]$$

S $\quad = e^{-t}\left[(2K_1 - 2K_2)\cos(t) + (-2K_1 - 2K_2)\sin(t)\right]$
$$+ te^{-t}\left[2K_2 \sin(t) - 2K_1 \cos(t)\right]$$

put $x_p(t)$ in diff equation

$$\frac{d^2 x_p}{dt^2} + 2\frac{d x_p}{dt} + 2x_p = e^{-t}\sin(t)$$

$$e^{-t}\left[(2K_1 - 2K_2)\cos(t) + (-2K_1 - 2K_2)\sin(t)\right] + te^{-t}\left[2K_2 \sin(t) - 2K_1 \cos(t)\right]$$
$$+ 2\left[e^{-t}\left[K_1 \sin(t) + K_2 \cos(t)\right] + te^{-t}\left[(K_1 - K_2)\cos(t) + (-K_1 - K_2)\sin(t)\right]\right]$$
$$+ 2\left[te^{-t}\left[K_1 \sin(t) + K_2 \cos(t)\right]\right] = e^{-t}\sin(t)$$

$$x_p(t) = -\frac{1}{2} t e^{-t} \cos(t)$$

$$\frac{dx_p}{dt} = -\frac{1}{2}\left[e^{-t}\cos(t) - te^{-t}\cos(t) - te^{-t}\sin t\right] = -\frac{1}{2}\left[e^{-t}\cos(t) - te^{-t}\left[\cos(t)+\sin(t)\right]\right]$$

$$\frac{d^2x_p}{dt^2} = -\frac{1}{2}\Big[-e^{-t}\cos(t) - e^{-t}\sin(t) - \Big[e^{-t}\left[\cos(t)+\sin(t)\right] - te^{-t}\left[\cos(t)+\sin(t)\right] + te^{-t}\left[-\sin(t)+\cos(t)\right]\Big]$$

$$= -\frac{1}{2}\left(-e^{-t}\left[\cos(t)+\sin(t)\right] - e^{-t}\left(\cos(t)+\sin(t)\right] + te^{-t}\,2\sin(t)\right]$$

$$= -\frac{1}{2}\left[-e^{-t}\left[2\cos(t)+2\sin(t)\right] + te^{-t}\,2\sin(t)\right] = \frac{1}{2}\left(e^{-t}\left(2\cos(t)+2\sin(t)\right] - te^{-t}2\sin(t)\right]$$

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = e^{-t}\sin(t)$$

$$\frac{1}{2}\left[e^{-t}\left[2\cos(t)+2\sin(t)\right] - 2te^{-t}\sin(t)\right] + 2\left(-\frac{1}{2}\right)\left[e^{-t}\cos(t) - te^{-t}\left[\cos(t)+\sin(t)\right]\right]$$

$$+2\left(-\frac{1}{2}te^{-t}\cos(t)\right] = e^{-t}\sin(t)$$

$$e^{-t}\left[\left[\cos(t)+\sin(t)\right] - \cos(t)\right]\Big] + te^{-t}\underbrace{\left(-\sin(t)+\cos(t)+\sin(t)-\cos(t)\right]}_{0} = e^{-t}\sin(t)$$

$$e^{-t}\sin(t) = e^{-t}\sin(t) \checkmark$$ {what we have found as $x_p(t)$ is correct}

$$e^{-t}\left[(2K_1-2K_2+2K_2)\cos(t)+(-2K_1-2K_2+2K_1)\sin(t)\right]$$

$$+\,t\,e^{-t}\left[\underbrace{\left[2K_2-2K_1-2K_2+2K_1\right]}_{0}\sin(t)+\underbrace{\left[-2K_1+2K_1-2K_2+2K_2\right]}_{0}\cos(t)\right]=e^{-t}\sin(t)$$

$$e^{-t}\left[2K_1\cos(t)+(-2K_2)\sin(t)\right]=e^{-t}\sin(t)$$

$K_1=0$

$K_2=-\frac{1}{2}$ //

(5)

$$x_p(t)=-\frac{1}{2}\,t\,e^{-t}\cos(t)$$

(1) $x_n(t) = e^{-t}\left[C_1 \sin(t) + C_2 \cos(t)\right] \longrightarrow$ char equation $s^2+2s+2=0 \Rightarrow s_{1,2} = -1 \pm j$

$$x(t) = x_n(t) + x_p(t) = e^{-t}\left[C_1 \sin(t) + C_2 \cos(t)\right] + \left(-\frac{1}{2}\right) t e^{-t} \cos(t)$$

$$x(0) = e^{-0}\left[C_1 \sin(0) + C_2 \cos(0)\right] + \left(-\frac{1}{2}\right)(0) e^{-0} \cos(0) = 1$$

$$C_2 = 1 \quad (1)$$

$$\left.\frac{dx}{dt}\right|_{t=0} = 1$$

$$\frac{dx}{dt} = -e^{-t}\left[C_1 \sin(t) + C_2 \cos(t)\right] + e^{-t}\left[C_1 \cos(t) - C_2 \sin(t)\right] + -\frac{1}{2}\left[e^{-t}\cos(t) - t e^{-t}\left[\cos(t) + \sin(t)\right]\right]$$

$$\left.\frac{dx}{dt}\right|_{t=0} = -e^{-0}\left[C_1 \sin(0) + C_2 \cos(0)\right] + e^{-0}\left[C_1 \cos(0) - C_2 \sin(0)\right] + -\frac{1}{2}\left[e^{-0}\cos(0) - 0\, e^{-0}\left[\cos(0) + \sin(0)\right]\right]$$

$$= -1\left[C_2\right] + 1\left[C_1\right] + -\frac{1}{2}\left[1\right] = 1 \quad \text{put } C_2 = 1$$

(1)

$$-1 \times 1 + C_1 - \frac{1}{2} = 1 \qquad C_1 = \frac{5}{2}$$

$$x(t) = e^{-t}\left[\frac{5}{2}\sin(t) + \cos(t)\right] - \frac{1}{2} t e^{-t} \quad (2)$$