ECE 233 Make-up
11-01-2013

Q-1-

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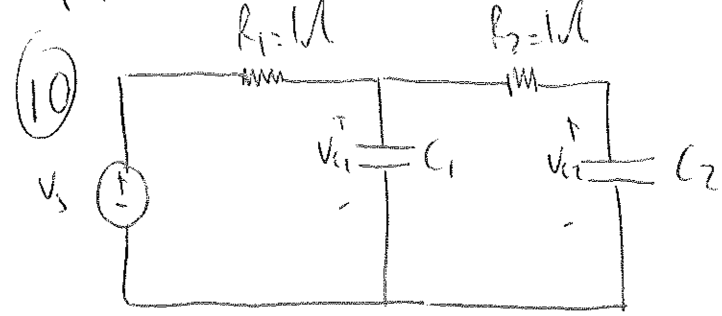

$$\frac{V_s - V_{c_1}}{R_1} = C_1 \frac{dV_{c_1}}{dt} + \frac{V_{c_1} - V_{c_2}}{R_2}$$

$R_1 = 1$
$R_2 = 1$

$$V_s - V_{c_1} = C_1 \frac{dV_{c_1}}{dt} + \frac{V_{c_1} - V_{c_2}}{1}$$

5 $$\boxed{\frac{dV_{c_1}}{dt} = \frac{V_s}{C_1} - \frac{2}{C_1} V_{c_1} + \frac{1}{C_1} V_{c_2}} \quad *$$

$$V_{c_1} = V_{R_2} + V_{c_2}$$

$$V_{c_1} = R_2 C_2 \frac{dV_{c_2}}{dt} + V_{c_2}$$

$$** \quad \boxed{\frac{dV_{c_2}}{dt} = \frac{V_{c_1}}{C_2} - \frac{V_{c_2}}{C_2}}$$

5

5

$$\frac{d^2V_{c_1}}{dt^2} = \frac{1}{C_1}\frac{dV_s}{dt} - \frac{2}{C_1}\frac{dV_{c_1}}{dt} + \frac{1}{C_1}\frac{dV_{c_2}}{dt}$$

3 $$\boxed{V_{c_2} = C_1 \frac{dV_{c_1}}{dt} + 2V_{c_1} - V_s}$$

$$\frac{d^2V_{c_1}}{dt^2} + \frac{2}{C_1}\frac{dV_{c_1}}{dt} = \frac{1}{C_1}\frac{dV_s}{dt} + \frac{1}{C_1}\left[\frac{V_{c_1}}{C_2} - \frac{V_{c_2}}{C_2}\right]$$

$$\frac{d^2V_{c_1}}{dt^2} + \frac{2}{C_1}\frac{dV_{c_1}}{dt} - \frac{1}{C_1C_2}V_{c_1} = \frac{1}{C_1}\frac{dV_s}{dt} - \frac{1}{C_1C_2}(V_{c_2})$$

$$\frac{d^2V_{c_1}}{dt^2} + \frac{2}{C_1}\frac{dV_{c_1}}{dt} - \frac{1}{C_1C_2}V_{c_1} = \frac{1}{C_1}\frac{dV_s}{dt} - \frac{1}{C_2}\frac{dV_{c_1}}{dt} - \frac{2}{C_1C_2}V_{c_1} + \frac{V_s}{C_1C_2}$$

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$$\frac{d^2V_{c_1}}{dt^2} + \left[\frac{2}{C_1} + \frac{1}{C_2}\right]\frac{dV_{c_1}}{dt} + \frac{1}{C_1C_2}V_{c_1} = \frac{1}{C_1}\frac{dV_s}{dt} + \frac{1}{C_1C_2}V_s$$

$\alpha = 1.5 \frac{rad}{sec}$ $\quad$ $w_0 = 1 \frac{rad}{sec}$

(3)

$s^2 + 2\alpha s + w_0^2 = 0$ $\quad$ $s^2 + \left[\frac{2}{C_1} + \frac{1}{C_2}\right] s + \frac{1}{C_1 C_2} = 0$

$\frac{2}{C_1} + \frac{1}{C_2} = 3$ $\quad$ $2C_2 + C_1 = 3C_1C_2$ $\quad$ $\frac{1}{C_1C_2} = 1$

$2C_2 + C_1 = 3$ $\quad$ $C_1 = \frac{1}{C_2}$

$2C_2 + \frac{1}{C_2} = 3$ $\quad$ $2C_2^2 - 3C_2 + 1 = 0$

(4)

$$C_2 = \frac{3 \mp \sqrt{9 - 4 \times 2 \times 1}}{4} = \frac{3 \mp 1}{4} = \begin{cases} 1 \rightarrow \boxed{C_2 = 1 \Rightarrow C_1 = 1} \\ \frac{1}{2} \rightarrow \boxed{C_2 = \frac{1}{2} \Rightarrow C_1 = 2} \end{cases}$$

Both solutions are possible

Q-2

(a) $\frac{dI_L}{dt} = V_s - V_{C_1} - V_{C_2}$ → This is a kind of Voltage-drop equation in a loop

$L\frac{dI_L}{dt}$ → $L = 1$ 3

$V_L = V_s - V_{C_1} - V_{C_2}$

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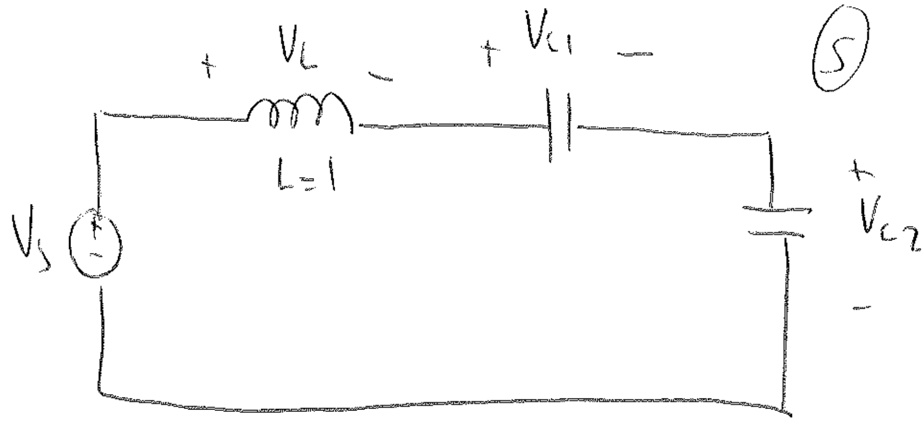
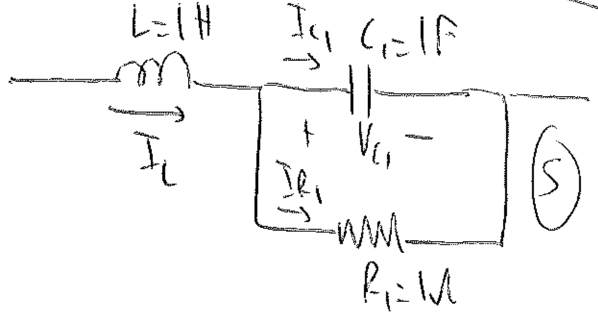


$\frac{dV_{C1}}{dt} = I_L - V_{C_1}$ → This is a current division equation

$C_1\frac{dV_{C_1}}{dt} = I_L - \frac{V_{C_1}}{1}$ 2 $C_1 = 1$ Farad 2 $R_1 = 1\Omega$

$I_L = I_{C_1} + I_{R_1}$ 2



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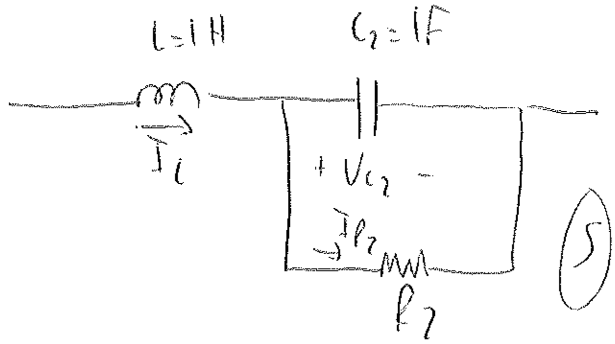
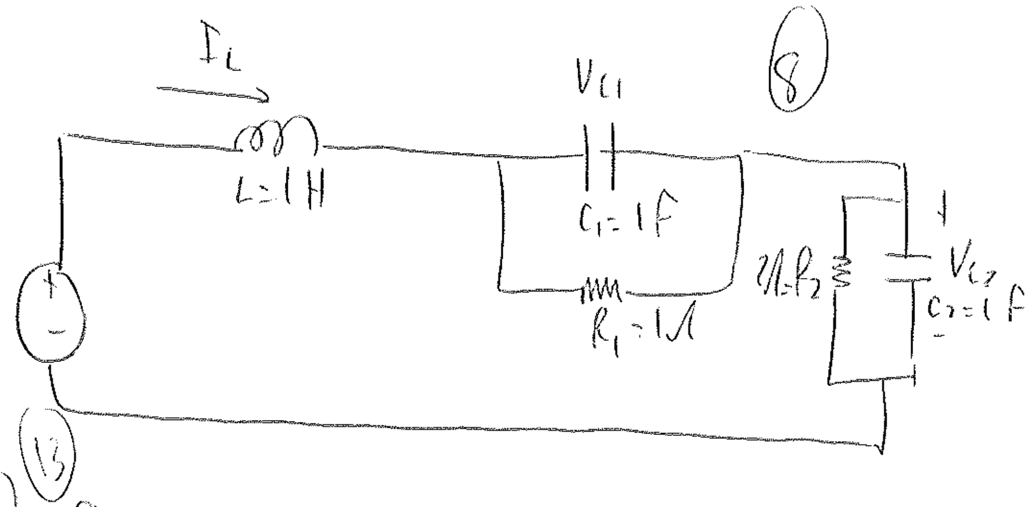
$\frac{dV_{C2}}{dt} = I_L - \frac{V_{C2}}{2}$ → This is a current division equation

$\underbrace{C_2\frac{dV_{C2}}{dt}}_{I_{C2}} = I_L - \underbrace{\frac{V_{C2}}{R_2}}_{I_{R2}}$ 2 $C_2 = 1F$ $R_2 = 2\Omega$ 2 $I_L = I_{C_2} + I_{R_2}$ 3



Fusing the circuits we will have the design



b

$$\begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_{C_1}}{dt} \\ \frac{dV_{C_2}}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}}_{A} \begin{bmatrix} I_L \\ V_{C_1} \\ V_{C_2} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} V_s$$

state-space representation

$$[sI - A] = \begin{bmatrix} s & 1 & 1 \\ -1 & s+1 & 0 \\ -1 & \frac{1}{2} & s+\frac{1}{2} \end{bmatrix}$$

$$\det\left(\left(sI - A\right)\right) = s(s+1)\left(s+\frac{1}{2}\right) - 1\left[(-1)\cdot\left(s+\frac{1}{2}\right)\right] + 1\left[0 - (-1)(s+1)\right]$$

$$= s(s+1)\left(s+\frac{1}{2}\right) + \left(s+\frac{1}{2}\right) + s + 1$$

$$= s^3 + \frac{3}{2}s^2 + \frac{1}{2}s + 2s + \frac{3}{2} = s^3 + \frac{3}{2}s^2 + \frac{5}{2}s + \frac{3}{2}$$

(c) $$\frac{d^2 I_L}{dt^2} = \frac{dV_s}{dt} - \frac{dV_{c_1}}{dt} - \frac{dV_{c_2}}{dt}$$

$$= \frac{dV_s}{dt} - \left(I_L - V_{c_1}\right) - \left(I_L - \frac{V_{c_2}}{2}\right)$$

$$\frac{d^2 I_L}{dt^2} = \frac{dV_s}{dt} + V_{c_1} + \frac{V_{c_2}}{2} - 2I_L \longrightarrow \frac{d^2 I_L}{dt^2}(0) = \frac{dV_s(0)}{dt} + V_{c_1}(0) + \frac{V_{c_2}(0)}{2} - 2I_L(0)$$

$$= 0 + 1 + \frac{1}{2} - 2 = \frac{3}{2} - 2 \; \frac{\text{Amp}}{\text{sec}^2}$$

$v_s(t) = 1\ \text{Volt}$

$$\frac{dV_s}{dt} = 0$$

I_L