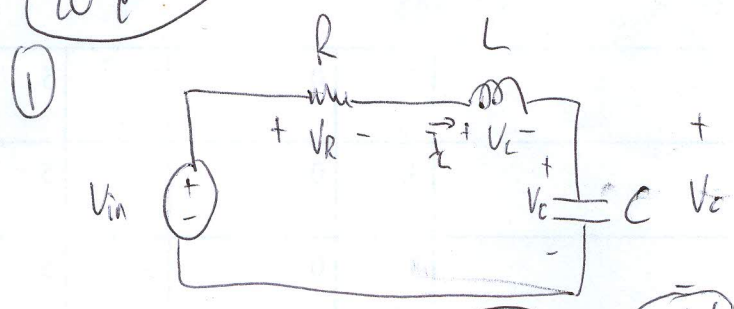


20 point



$$V_{in} = V_R + V_L + V_C$$

$$V_{in} = R I_L + L \frac{dI_L}{dt} + V_C$$

$$\frac{dI_L}{dt} = \frac{V_{in}}{L} - \frac{R}{L} I_L - \frac{V_C}{L}$$

$$C \frac{dV_C}{dt} = I_L$$

$$\frac{dV_C}{dt} = \frac{I_L}{C}$$

Take derivative of (2) put (1) inside

$$\frac{d^2 I_L}{dt^2} = \frac{1}{L} \frac{dV_{in}}{dt} - \frac{R}{L} \frac{dI_L}{dt} - \frac{1}{L} \frac{dV_C}{dt}$$

$$\frac{d^2 I_L}{dt^2} + \frac{R}{L} \frac{dI_L}{dt} = \frac{1}{L} \frac{dV_{in}}{dt} - \frac{1}{L} \frac{I_L}{C} \rightarrow \frac{d^2 I_L}{dt^2} + \frac{R}{L} \frac{dI_L}{dt} + \frac{1}{LC} I_L = \frac{1}{L} \frac{dV_{in}}{dt}$$

Take derivative of (1) put (2) inside

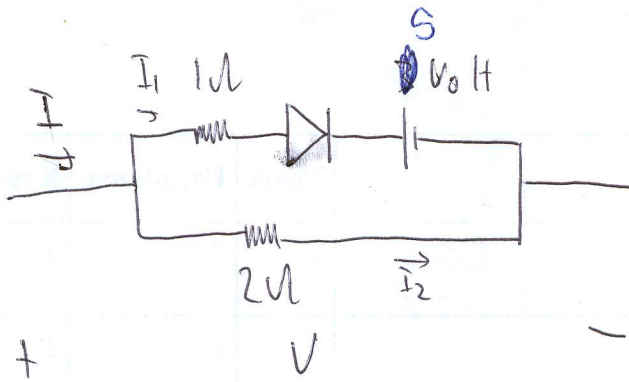
$$\frac{d^2 V_C}{dt^2} = \frac{1}{C} \frac{dI_L}{dt} = \frac{1}{C} \left[\frac{V_{in}}{L} - \frac{R}{L} I_L - \frac{1}{L} V_C \right]$$

$$\frac{d^2 V_C}{dt^2} + \frac{1}{LC} V_C + \frac{R}{CL} I_L = \frac{V_{in}}{LC}$$

\downarrow
 $C \frac{dV_C}{dt}$

$$\frac{d^2 V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{1}{LC} V_C = \frac{1}{LC} V_{in}$$

②



if ~~resistor~~ $V > S \rightarrow I_1 \checkmark$
 $\rightarrow I_2 \checkmark$

if $V < S \rightarrow I_1 \times$
 $\rightarrow I_2 \checkmark$

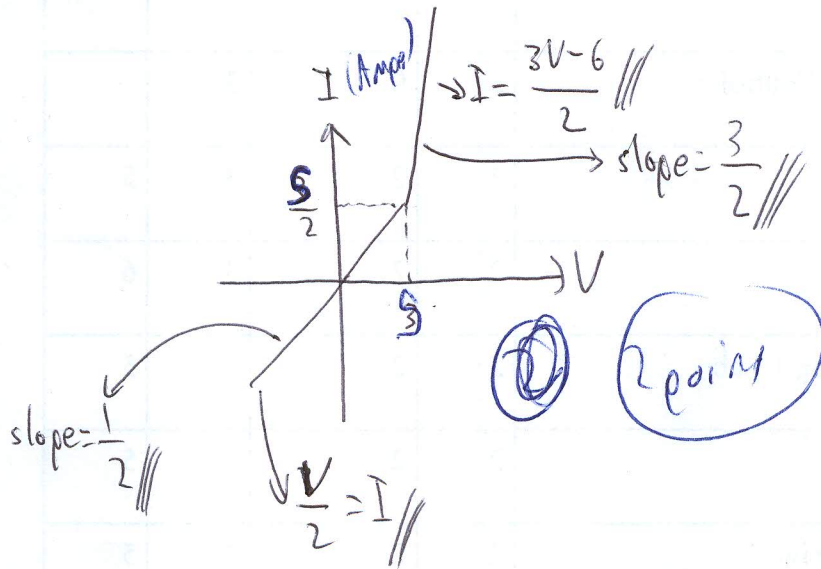
$$I = I_1 + I_2 \leq \frac{V - S}{1} + \frac{V}{2}$$

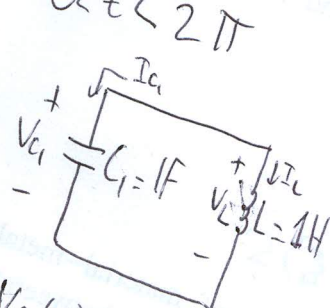
$$= \frac{3V - 6}{2}$$

$$I = I_1 + I_2 = I_2 = \frac{V}{2}$$

↓
0

3 point





$V_{c1}(0) = 1 \text{ Volt}$
 $I_L(0) = 0 \text{ Amper}$

$V_{c1} \neq V_L$
 $V_{c1} = L \frac{dI_L}{dt}$
 $I_{c1} + I_L = 0$
 $C_1 \frac{dV_{c1}}{dt} + I_L = 0$

① $\frac{dI_L}{dt} = V_{c1}$

① $\frac{dV_{c1}}{dt} = -I_L$

$\frac{d^2 I_L}{dt^2} = \frac{dV_{c1}}{dt} = -I_L$

$\frac{d^2 I_L}{dt^2} + I_L = 0$ ②

$\frac{d^2 V_{c1}}{dt^2} = -\frac{dI_L}{dt} = -V_{c1}$

$\frac{d^2 V_{c1}}{dt^2} + V_{c1} = 0$ ②

char-egu ② $s^2 H = 0$

② $s = \pm j$

$V_{ch}(t) = A \sin(t) + B \cos(t)$

~~.....~~
 $I_{ch}(t) = K \sin(t) + M \cos(t)$

$V_{c1}(0) = \frac{A \sin(0)}{0} + \frac{B \cos(0)}{1} = 1$
 $B = 1$

$\frac{dV_{c1}}{dt}(0) = -I_L(0) = 0 = A \cos(0) - B \sin(0)$
 $A = 0$

Since there is no input $V_{cip} = 0, I_{cp} = 0$
 $V_{c1} = V_{ch} + V_{cip} = A \sin(t) + B \cos(t)$

$I_L = I_{ch} + I_{cp} = K \sin(t) + M \cos(t)$

$\frac{dV_{c1}}{dt} = A \cos(t) - B \sin(t)$

$(t) = \cos(t) \quad 0 < t < 2\pi$

$\frac{dV_{c1}}{dt} = -\frac{d}{dt} \cos(t) = \sin(t) \quad 0 < t < 2\pi$

Gides ②

Amper

= 0 Amper no change
are connected in parallel when $t=2\pi$ so the
the capacitors will be preserved.

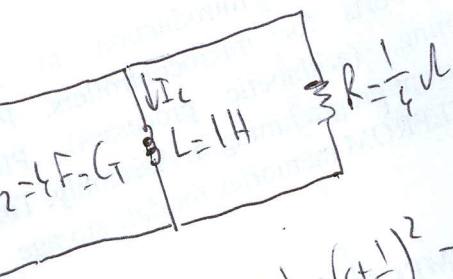
$Q(2\pi^+)$

$$V_{C_2}(2\pi^+) = (C_1 + C_2) V_{C_1}(2\pi^+)$$

$3 \times 5 = 4 \times V_{C_1}(2\pi^+)$

(3)

$$V_{C_1}(2\pi^+) = 4 \text{ Volt}$$



diff-equations

$$\frac{d^2 I_L}{dt^2} + \frac{1}{RC} \frac{dI_L}{dt} + \frac{1}{LC} I_L = 0$$

$$\frac{d^2 V_{C_1}}{dt^2} + \frac{1}{RC} \frac{dV_{C_1}}{dt} + \frac{1}{LC} V_{C_1} = 0$$

(3)

$$V_{C_1}(t) = (A + Bt) e^{-\frac{1}{2}t} \text{ or } V_{C_1}(t) = (K + M(t-2\pi)) e^{-\frac{1}{2}(t-2\pi)}$$

(2)

$$\text{r-equation} = s^2 + st + \frac{1}{4} = \left(s + \frac{1}{2}\right)^2$$

(2)

we also know that
 $I_C + I_L + I_R = 0$

$$C \frac{dV_{C_1}}{dt} + I_L + \frac{V_{C_1}}{\frac{1}{4}} = 0$$

$$\frac{dV_{C_1}}{dt} = -\frac{I_L}{C} - \frac{4V_{C_1}}{C} = -\frac{I_L}{4} - V_{C_1}$$

(2)

$$V_{C_1}(2\pi^+) = (K + M \times 0) e^{-\frac{1}{2}(2\pi-2\pi)} = K = 4$$

$$\frac{dV_{C_1}}{dt}(2\pi^+) = -\frac{I_L(2\pi^+)}{4} - V_{C_1}(2\pi^+) = 0 - 4 = -4$$

$$V_{C_1} = (4 - 2(t-2\pi)) e^{-\frac{1}{2}(t-2\pi)}$$

(2)

$$\frac{dV_{C_1}}{dt} = (K + M(t-2\pi)) \left(-\frac{1}{2}\right) e^{-\frac{1}{2}(t-2\pi)} + M e^{-\frac{1}{2}(t-2\pi)}$$

$$\frac{dV_{C_1}}{dt}(2\pi^+) = -\frac{1}{2}K + M = -4$$

$$-\frac{1}{2} \times 4 + M = -4$$

$$M = -2$$

(1)

Q- $\frac{d^2 x}{dt^2} + x = \sin(t) = V_{in}(t)$

find $x_p(t) =$

char equation $s^2 + 1 = 0 \rightarrow x_h(t) = A \sin(t) + B \cos(t)$
 than

$s_{1,2} = \pm j$

$V_{in}(t) = \sin(t) \rightarrow \omega = 1 \text{ rad/sec}$

↳ angular frequency of the input

$s_1 = j = j \times \omega$
 $= j \times 1 =$

(The natural frequency of the char equation $j = j \times \omega$ (j multiplied by input angular frequency)

so $x_p(t) = A t \sin(t) + B t \cos(t) = t [A \sin(t) + B \cos(t)]$

$\frac{d^2}{dt^2} x_p(t) + x_p(t) = \sin(t)$

⑤ $\frac{d}{dt} x_p(t) = [A \sin(t) + B \cos(t)] + t [A \cos(t) - B \sin(t)]$

⑦ $\frac{d^2}{dt^2} x_p(t) = [A \cos(t) - B \sin(t)] + [A \cos(t) - B \sin(t)] + t [-A \sin(t) - B \cos(t)]$

⑥ $2A \cos(t) - 2B \sin(t) + t [-A \sin(t) - B \cos(t)] + t [A \sin(t) + B \cos(t)] = \sin(t)$

⑧ $2A \cos(t) - 2B \sin(t) = \sin(t) \quad A=0 \quad B = -\frac{1}{2}$

$x_p(t) = -\frac{1}{2} t \cos(t)$

④