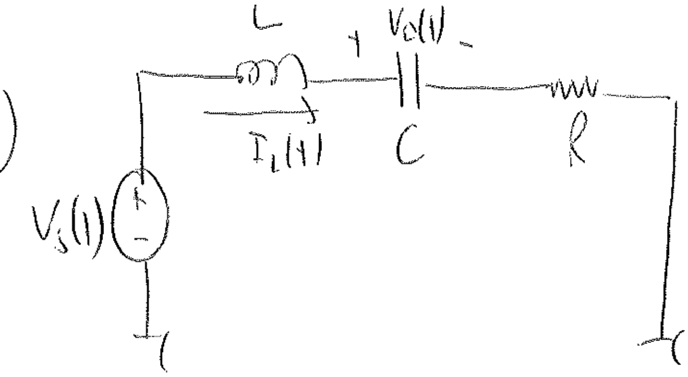
ECE 233
Final
11-01-2013

Q-1-

(a)



$$C\frac{dV_c}{dt} = I_c = I_L \qquad \boxed{\frac{dV_c}{dt} = \frac{1}{C} I_L} \quad (4)$$

$$V_s = V_L + V_c + V_R \qquad V_s = L\frac{dI_L}{dt} + V_c + R I_L$$

$$\boxed{\frac{dI_L}{dt} = \frac{V_s}{L} - \frac{V_c}{L} - \frac{R}{L} I_L} \quad (4)$$

$$(2) \begin{bmatrix} \frac{dI_L}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_s$$

(b)

$$(1) \quad \frac{d^2V_c}{dt^2} = \frac{1}{C}\frac{dI_L}{dt} = \frac{1}{C}\left[\frac{V_s}{L} - \frac{V_c}{L} - \frac{R}{L} I_L\right] \quad (2)$$

girdi (2)

$$\frac{d^2V_c}{dt^2} + \frac{1}{CL} V_c = \frac{1}{LC} V_s - \frac{R}{LC} I_L \rightarrow C\frac{dV_c}{dt}$$

$$(1) \quad \boxed{\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{1}{LC} V_c = \frac{1}{LC} V_s}$$

$$\frac{d^2 I_L}{dt^2} = \frac{1}{L}\frac{dV_s}{dt} - \frac{1}{L}\frac{dV_c}{dt} - \frac{R}{L}\frac{dI_L}{dt} \quad (2)$$

$$\frac{d^2 I_L}{dt^2} + \frac{R}{L}\frac{dI_L}{dt} = \frac{1}{L}\frac{dV_s}{dt} - \frac{1}{L}\frac{dV_c}{dt} \Rightarrow \frac{I_L}{C} \quad (1)$$

$$\boxed{\frac{d^2 I_L}{dt^2} + \frac{R}{L}\frac{dI_L}{dt} + \frac{1}{LC} I_L = \frac{1}{L}\frac{dV_s}{dt}} \quad (4)$$

Q-2- $\frac{dV_c}{dt} = \delta(t) - 2I_L(t) - V_c(t)$

These integrals $= 0$ since $I_L(t)$ and are continuous o

$$\int_{I_L(0^-)}^{I_L(0^+)} dV_c = \int_{0^-}^{0^+} \delta(t')\,dt' - 2\int_{0^-}^{0^+} I_L(t')\,dt' - \int_{0^-}^{0^+} V_c(t')\,dt'$$

$$V_c(0^+) - V_c(0^-) = 1 - 0 - 0$$

$V_c(0^+) = V_c(0^-) + 1$

$V_c(0^+) = 1 + 1 = 2\,V_o$

$$\frac{dI_L}{dt} = -2V_c \qquad \int_{I_L(0^-)}^{I_L(0^+)} dI_L = -2\int_{0^-}^{0^+} V_c(t')\,dt' \Rightarrow I_L(0^+) - I_L(0^-) = 0$$

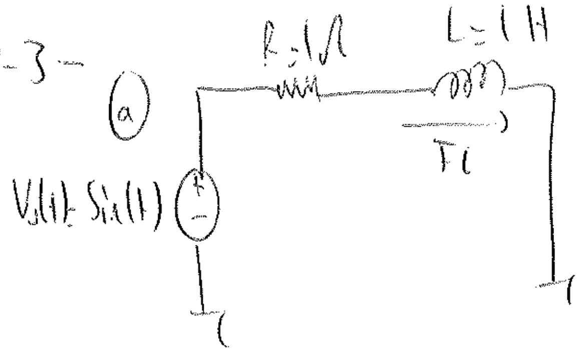
$I_L(0^+) = I_L(0^-) = $

$$\frac{dI_L}{dt}(0^+) = -2V_c(0^+) = -2\times 2 = -4\ \frac{\text{Amper}}{\text{sec}} \quad (1.5)$$

$$\frac{dV_c}{dt}(0^+) = \delta(0^+) - 2I_L(0^+) - V_c(0^+) = 0 - 2\times 1 - 2 = -4\ \frac{\text{Volt}}{\text{sec}}$$

(1.5

Q-3 — (a)



$V_s = V_R + V_L$ (2)

$Sin(t) = 1 \times I_L + 1 \dfrac{dI_L}{dt}$ (1) (1)

(1) $\boxed{\dfrac{dI_L}{dt} + I_L = Sin(t)}$ $t \geq 0$

(5) (b) $I_{Lp} = K_1 Sin(t) + K_2 Cos(t)$ (1)

(2)

$\dfrac{d}{dt}\left(K_1 Sin(t) + K_2 Cos(t)\right) + K_1 Sin(t) + K_2 Cos(t) = Sin(t)$

$K_1 Cos(t) - K_2 Sin(t) + K_1 Sin(t) + K_2 Cos(t) = Sin(t)$

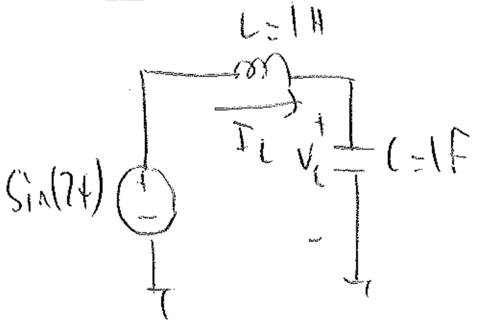
$\left(K_1 - K_2\right) Sin(t) + \left(K_1 + K_2\right) Cos(t) = Sin(t)$

(1)

$K_1 - K_2 = 1$ $\quad K_1 + K_2 = 0$ $\quad K_1 = \frac{1}{2}$ $\quad K_2 = -\frac{1}{2}$

$I_{Lp} = \frac{1}{2} Sin(t) - \frac{1}{2} Cos(t)$ (1)

Q-4 $\quad 0 \le t < \pi \longrightarrow V_s(t) = \sin(2t)$



$V_{C_2}(\pi^-) = V_{C_2}(0) = \frac{4}{3}$ Volt

(no change since it is not active when $0 \le t < \pi$)

$$\sin(2t) = V_L + V_C = 1\frac{dI_L}{dt} + V_C \qquad \boxed{\frac{dI_L}{dt} + V_C = \sin(2t)} \; 1 \qquad \boxed{1\frac{dV_C}{dt} = I_L} \; 1$$

2

$$\frac{d^2 I_L}{dt^2} + \frac{dV_C}{dt} = 2\cos(2t) \Rightarrow \boxed{\frac{d^2 I_L}{dt^2} + I_L = 2\cos(2t)}$$

$$\frac{d^2 V_C}{dt^2} = \frac{dI_L}{dt} = \sin(2t) - V_C$$

$$\boxed{\frac{d^2 V_C}{dt^2} + V_C = \sin(2t)} \; 2$$

char-equ: $s^2 + 1 = 0$ 1 $\quad s_{1,2} = \mp j \rightarrow$ natural frequencies 1

$V_{ch} = K_1 \sin(t) + K_2 \cos(t)$ 1

$V_{cp} = M_1 \sin(2t) + M_2 \cos(2t)$ 1

$V_C(0) = 0 \quad I_L(0) = 0$

$\frac{dV_C}{dt}(0) = I_L(0) = 0$ 1

1

$$\frac{d^2}{dt^2}\left(M_1 \sin(2t) + M_2 \cos(2t)\right) + M_1 \sin(2t) + M_2 \cos(2t) = \sin(2t)$$

$$-4M_1 \sin(2t) - 4M_2 \cos(2t) + M_1 \sin(2t) + M_2 \cos(2t) = \sin(2t)$$

$$-3M_1 \sin(2t) - 3M_2 \cos(2t) = \sin(2t) \qquad M_2 = 0 \quad M_1 = \frac{-1}{3}$$

$V_{cp} = -\frac{1}{3}\sin(2t)$ 2

$$V_C = V_{cp} + V_{ch} = K_1 \sin(t) + K_2 \cos(t) - \frac{1}{3}\sin(2t)$$

$$V_C(0) = K_1 \sin(0) + K_2 \cos(0) - \frac{1}{3}\sin(0) = K_2 = 0 \qquad V_C(t) = K_1 \sin(t) - \frac{1}{3}\sin(2t)$$

$$\frac{dV_C}{dt} = K_1 \cos(t) - \frac{2}{3}\cos(2t) \qquad \frac{dV_C}{dt}(0) = K_1 - \frac{2}{3} = 0 \qquad K_1 = \frac{2}{3}$$

$V_c(t) = \frac{2}{3}\sin(t) - \frac{1}{3}\sin(2t)$ (3) $\quad 0 \le t < \pi$

$C\frac{dV_c}{dt} = I_c = \frac{dV_c}{dt} = \frac{2}{3}\cos(t) - \frac{2}{3}\cos(2t)$ (1) $\quad 0 \le t < \pi$

$V_c(\pi^-) = 0$ Volt (1)

$I_c(\pi^-) = -\frac{2}{3} - \frac{2}{3} = -\frac{4}{3}$ ~~Volt~~ Amper (1)

$t = \pi$ (1)

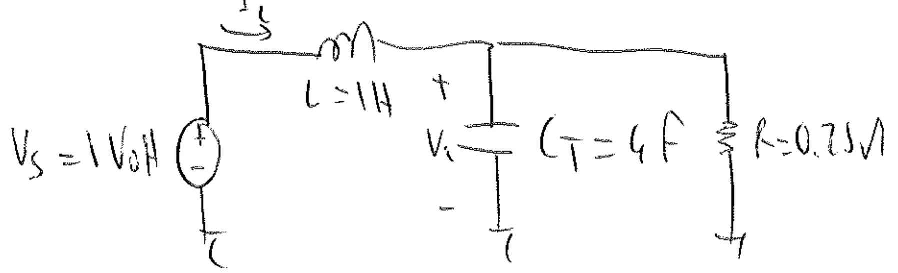
$Q(\pi^-) = Q(\pi^+) \qquad C_1 V_c(\pi^-) + C_2 V_{c_2}(\pi^-) = (C_1 + C_2)\, V_c(\pi^+)$

$1 \times 0 + 3 \times \frac{4}{3} = 4 \quad V_c(\pi^+)$

$\boxed{V_c(\pi^+) = 1 \text{ Volt}}$ (1)

No change in I_L hence $I_L(\pi^-) = I_L(\pi^+) = \cancel{-}\frac{4}{3}$ Amper

$t > \pi$



$V_s = V_L + V_c \qquad \boxed{\frac{dI_L}{dt} = 1 - V_c}$ (1)

$1 = 1\frac{dI_L}{dt} + V_c$

$I_L = I_C + I_R$

$I_L = C\frac{dV_c}{dt} + \frac{V_c}{R} \qquad \boxed{I_L = 4\frac{dV_c}{dt} + 4V_c}$ (1)

$$\frac{dI_L}{dt} = 4\frac{d^2V_c}{dt^2} + 4\frac{dV_c}{dt} = 1 - V_c \Rightarrow \boxed{\frac{d^2V_c}{dt^2} + \frac{dV_c}{dt} + \frac{1}{4}V_c = \frac{1}{4}}$$ (1)

$V_c(\Pi^+) = 1$ Volt

$$\frac{dV_c}{dt}(\Pi^+) = \frac{1}{4}I_L(\Pi^+) - V_c(\Pi^+)$$

$$= \frac{1}{4} \times \left(-\frac{4}{3}\right) - 1 = -\frac{4}{3}$$ (1)

char-equation $s^2 + s + \frac{1}{4} = 0$ (1)

natural frequencies $s_{1,2} = -\frac{1}{2}$ (1)

$$V_{ch} = \left[K_1 + K_2(t-\Pi)\right]e^{-\frac{1}{2}(t-\Pi)}$$ (1)

$V_{cp} = M$ (0.5)

$$\frac{d^2}{dt^2}M + \frac{d}{dt}M + \frac{1}{4}M = \frac{1}{4}$$ $M = 1$ (1)

Gidiş 8

$$V_c(t) = 1 + \left[K_1 + K_2(t-\Pi)\right]e^{-\frac{1}{2}(t-\Pi)}$$ (0.5)

$$V_c(\Pi^+) = 1 = 1 + \left[K_1 + K_2(\Pi-\Pi)\right]e^{-\frac{1}{2}(\Pi-\Pi)}$$

$$K_1 = 0$$

$$V_c(t) = 1 + K_2(t-\Pi)e^{-\frac{1}{2}(t-\Pi)}$$

$$\frac{dV_c}{dt} = K_2\left[e^{-\frac{1}{2}(t-\Pi)} + (t-\Pi)\left(-\frac{1}{2}\right)e^{-\frac{1}{2}(t-\Pi)}\right]$$

$$\frac{dV_c}{dt}(\Pi^+) = K_2\left[1+0\right] = -\frac{4}{3} \qquad K_2 = -\frac{4}{3}$$

(1) $V_c(t) = 1 + \left(-\frac{4}{3}\right)(t-\Pi)e^{-\frac{1}{2}(t-\Pi)}$ $\quad \frac{dV_c}{dt} = \left(-\frac{4}{3}\right)\left[e^{-\frac{1}{2}(t-\Pi)} + (t-\Pi)\left(-\frac{1}{2}\right)e^{-\frac{1}{2}(t-\Pi)}\right]$

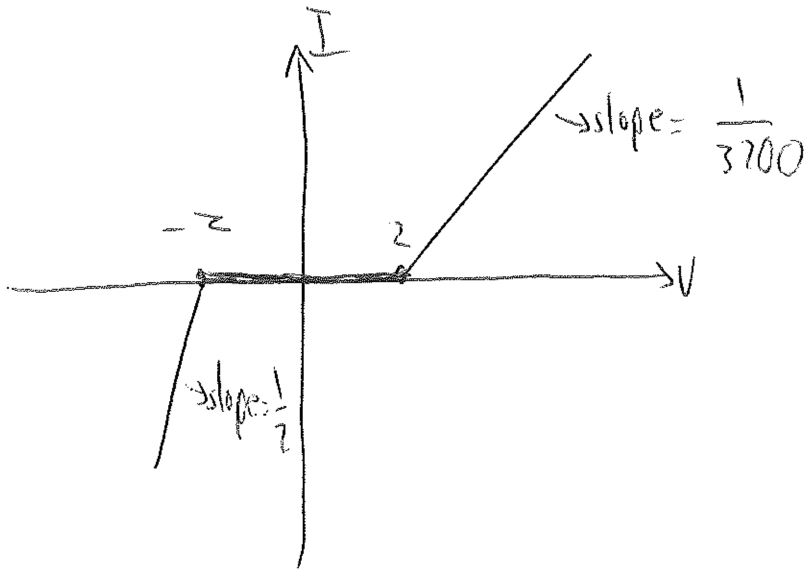
$$I_L(t) = 4\frac{dV_c}{dt} + 4V_c = 4\left[\frac{dV_c}{dt} + V_c\right] = 4\left[\left(-\frac{4}{3}\right)\left[e^{-\frac{1}{2}(t-\Pi)} + (t-\Pi)\left(-\frac{1}{2}\right)e^{-\frac{1}{2}(t-\Pi)}\right] + 1 + \left(-\frac{4}{3}\right)(t-\Pi)e^{-\frac{1}{2}(t-\Pi)}\right]$$

Q-5 -

$V > V_a \qquad I = \dfrac{V - V_a}{R_1 + R_3} = \dfrac{V - 2}{3200\,\Omega}$ (8)

$V_b < V < V_a \qquad I = 0$ (6)

$V < V_b \qquad I = -\left[\dfrac{V_b - V}{R_3 + R_2}\right] = \dfrac{V - V_b}{2000\,\Omega} = \dfrac{V + 2}{2000}$ (6)



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