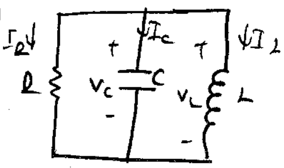
24.07.2010

(ii)



$C = \frac{1}{2}F \quad L = \frac{1}{8}H$

$I_C(0) = -4A$

$V_C(0) = 5V$

$R = \frac{1}{4}$

$2\alpha = \frac{1}{RC}$

$\alpha = \frac{1}{2RC} = \frac{1}{2\times\frac{1}{4}\times\frac{1}{2}} = 4$

$\omega_0 = \sqrt{\frac{1}{LC}} = 4 \text{ rad/sec}$

$\alpha = \omega_0 = 4$

$s^2 + 2\alpha s + \omega_0^2 = 0$ (char eqn)

$s^2 + 8s + 16 = 0$

$s_{1,2} = -4 = s_0$ (critically damped)

$V_C(t) = (A_1 + A_2 t)\, e^{-4t}$

$I_L(t) = (B_1 + B_2 t)\, e^{-4t}$

$I_L(0) = -4 = B_1$

$$\frac{dI_L}{dt} = B_2 e^{-4t} + -4e^{-4t}(B_1 + B_2 t)$$

$$\left.\frac{dI_L}{dt}\right|_{t=0} = B_2 - 4B_1 = 40$$

$$\left.\frac{dI_C}{dt}\right|_{t=0} = \frac{V_C(0)}{L} = \frac{5}{1/8} = 40$$

$B_2 - 4B_1 = 40$

$B_2 - 4(-4) = 40$

$B_2 = 24$

$L\frac{dI_L}{dt} = V_L \qquad \frac{dI_L}{dt} = \frac{V_L}{L} = \frac{V_C}{L}$

$I_L(t) = (-4 + 24t)\, e^{-4t}$

$L\frac{dI_L}{dt} = V_C$

$$= \frac{1}{8}\left[\frac{d}{dt}(-4 + 24t)e^{-4t}\right]$$

$$= \frac{1}{8}\left[24e^{-4t} + -4e^{-4t}(-4 + 24t)\right]$$

$$= \frac{1}{8}\left[24e^{-4t} + 16e^{-4t} - 96te^{-4t}\right] = [5 - 12t]e^{-4t}$$

$V_C(t) = (5-12t)e^{-4t}$

$V_C(0) = 5$

$\lim_{t\to\infty} V_C(t) = 0$

$V_C(t_x) = 0$

$(5-12t_x)e^{-4t_x} = 0 \Rightarrow t_x = \frac{5}{12}$

$\frac{dV_C}{dt}(0) = -\frac{V_0}{RC} - \frac{I_0}{C}$

$= -\frac{5}{\frac{1}{4}\times\frac{1}{2}} - \frac{(-4)}{\frac{1}{2}} = -32$

$\frac{dV_C}{dt} = -12e^{-4t} + (5-12t)(-4)e^{-4t}$

$= -32e^{-4t} + 48te^{-4t}$

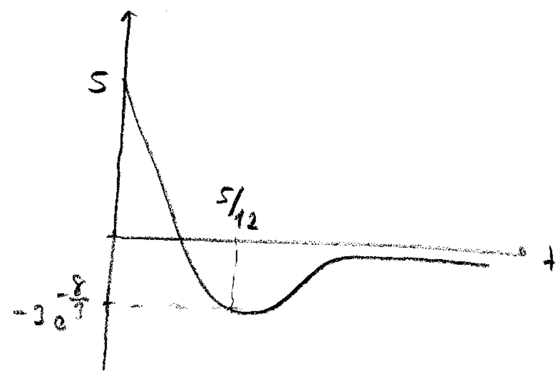
$= (-32 + 48t)e^{-4t}$

$\left.\frac{dV_C}{dt}\right|_{t_k} = 0 = (-32 + 48t_k)e^{-4t_k}$

$t_k = \frac{2}{3}$

$V_C(t_k) = \left(5 - 12\times\frac{2}{3}\right)e^{-4\frac{2}{3}}$

$= -3e^{-\frac{8}{3}}$



(ii) $R = \frac{1}{3}$ $\quad s^2 + 2\alpha s + \omega_o^2 = 0$ (char eq)

$$2\alpha = \frac{1}{RC} = \frac{1}{\frac{1}{3} \times \frac{1}{2}} = 6$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{\frac{1}{2} \times \frac{1}{8}} = 16$$

$\omega_o = 4$ rad/sec

$\alpha = 3$ $\quad \alpha < \omega_o$

$$s^2 + 6s + 16 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$= -3 \pm \sqrt{9 - 16}$$

$= -3 \mp j\sqrt{7}$ (complex conjugate root pairs) (underdamped case)

$$V_C(t) = e^{-3t}\left[A_1 \cos(\sqrt{7}t) + A_2 \sin(\sqrt{7}t)\right]$$

$$I_L(t) = e^{-3t}\left[B_1 \cos(\sqrt{7}t) + B_2 \sin(\sqrt{7}t)\right]$$

$$\boxed{I_L(0) = -4 = B_1}$$

$$\frac{dI_L}{dt} = -3e^{-3t}\left[B_1 \cos(\sqrt{7}t) + B_2 \sin\sqrt{7}t\right] + e^{-3t}\left[-\sqrt{7}B_1 \sin(\sqrt{7}t) + \sqrt{7}B_2 \cos(\sqrt{7}t)\right]$$

$$\left.\frac{dI_L}{dt}\right|_{t=0} = -3B_1 + \sqrt{7}B_2 \longrightarrow -3(-4) + \sqrt{7}\,B_2 = 40$$

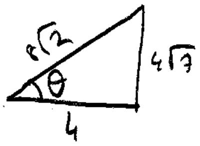
$$\boxed{B_2 = \frac{28}{\sqrt{7}} = 4\sqrt{7}}$$

$$\left.\frac{dI_L}{dt}\right|_{t=0} = \frac{V_C(0)}{L} = \frac{5}{\frac{1}{8}} = 40$$

$$I_L(t) = e^{-3t}\left[-4\cos(\sqrt{7}t) + 4\sqrt{7}\cos(\sqrt{7}t)\right]$$

$L\frac{dI_L}{dt} = V_C(t)$ $\quad$ HW

(iii-2) $|x| = \sqrt{(-4)^2+(4\sqrt{7})^2} = \sqrt{16+16\times 7} = 8\sqrt{2}$



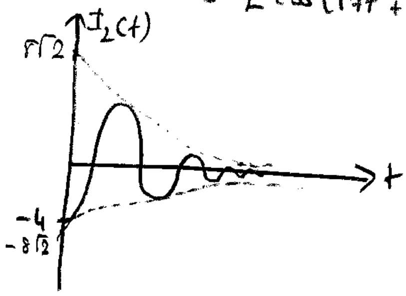
$$I_L(t) = e^{-3t}\left[\frac{-4}{8\sqrt{2}}\cos(\sqrt{7}t) + \frac{4\sqrt{7}}{8\sqrt{2}}\sin(\sqrt{7}t)\right] 8\sqrt{2}$$

$$= e^{-3t}\left[\cos(\sqrt{7}t)\cos(\theta) + \sin(\theta)\sin(\sqrt{7}t)\right]$$

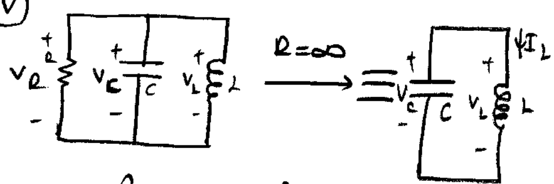
$$= -8\sqrt{2}\, e^{-3t}\left[\cos(\sqrt{7}t)\cos(\theta) - \sin(\sqrt{7}t)\sin\theta\right]$$

$$= -8\sqrt{2}\, e^{-3t}\left[\cos(\sqrt{7}t+\theta)\right]$$

$$V_C(t) = -8\sqrt{2}\, e^{-3t}\left[\cos(\sqrt{7}t + \tan^{-1}(\sqrt{7}))\right]$$



(iv)



$s^2 + 2\alpha s + w_0^2$

$\alpha = \frac{1}{2RC} = 0$ $\qquad s^2+16=0$ (char eq)

$w_0 = \sqrt{\frac{1}{LC}} = 4$ $\qquad s_{1,2} = \mp 4j$ (natural frequencies)

$V_C(t) = A_1\cos(4t) + A_2\sin(4t)$

$I_L(t) = B_1\cos(4t) + B_2\sin(4t)$

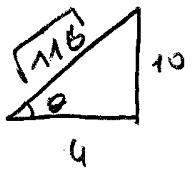
$\boxed{I_L(0) = -4 = B_1}$

$\frac{dI_L}{dt} = -4B_1\sin(4t) + 4B_2\cos(4t)$

$\left.\frac{dI_L}{dt}\right|_{t=0} = 4B_2$

$\left.\frac{dI_L}{dt}\right|_{t=0} = \frac{V_C(0)}{L} = \frac{5}{1/8} = 40$ $\qquad 4B_2 = 40$ $\qquad \boxed{B_2 = 10}$

$I_L = -4\cos(4t) + 10\sin(4t)$



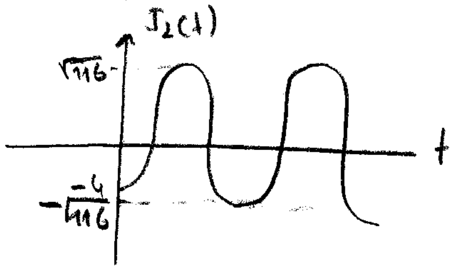
$$I_L(t) = \sqrt{116}\left[\frac{-4}{\sqrt{116}}\cos(4t) + \frac{10}{\sqrt{116}}\sin(4t)\right]$$

$$= -\sqrt{116}\left[\cos(\theta)\cos(4t) - \sin\theta\sin(4t)\right]$$

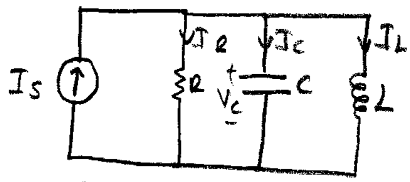
$$= \sqrt{-116}\cos(4t+\theta)$$

$$I_L(t) = -\sqrt{116}\cos\left(4t + \tan^{-1}\frac{5}{2}\right)$$

$$L\frac{dI_L}{dt} = V_c \quad \text{HW}$$



Complete Response



$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + w_o^2 V_c = \frac{1}{C}\frac{dI_s}{dt}$$

$$V_c(t_o) = V_o \qquad \frac{dV_c}{dt}(t_o) = -2\alpha V_o - \frac{I_o}{C} + \frac{1}{C}I_s(t_o)$$

$$\frac{d^2I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + w_o^2 I_L = w_o^2 I_s$$

$$I_L(t_o) = I_o \qquad \frac{dI_L}{dt}(t_o) = \frac{V_o}{L}$$

$$V_c(t) = \underbrace{V_{ch}(t)}_{\text{Homogeneous solution}} + \underbrace{V_{cp}(t)}_{\text{Particular solution}}$$

OR

$$V_c(t) = \underbrace{V_{czi}(t)}_{\text{zero input solution}} + \underbrace{V_{czs}(t)}_{\text{zero state solution}}$$

$t_0 = 0$

Contant input $= I_s(t) = I_s$
(DC Input)

$$\frac{d^2 V_c}{dt^2} + 2\alpha \frac{dV_c}{dt} + w_0^2 V_c = \frac{1}{c} \underbrace{\frac{d}{dt} I_s}_{0} = 0$$

$V_c(t_0) = V_0 \quad \frac{dV_c}{dt}(t_0) = -2\alpha V_0 - \frac{I_0}{c} + \frac{I_s}{c}$

$s^2 + 2\alpha s + w_0^2 = 0$ → critically damped $\} V_{ch} = (A_1 + A_2 t) e^{s_0 t}$
→ under damped
→ over damped
→ purely sinusoidal $\} V_{ch} = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

overdamped
underdamped
purely sinusoidal $\} A_1 e^{s_1 t} + A_2 e^{s_2 t}$

overdamped → $A_1 e^{s_1 t} + A_2 e^{s_2 t} = V_{ch}(t)$

Underdamped → $s_1 = s_2^* \quad s_1 = a_1 + ja_2$

$$V_{ch}(t) = e^{a_1 t}\left[B_1 \cos(a_2 t) + B_2 \sin(a_2 t)\right]$$

purely sinusoidal → $s_1 = s_2^* \quad s_1 = +ja_2$

$$V_{ch} = \left[D_1 \cos(a_2 t) + D_2 \sin(a_2 t)\right]$$

if system overdamped ;

$$V_{ch} = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\frac{d^2 V_c}{dt^2} + 2\alpha \frac{dV_c}{dt} + w_0^2 V_c = \frac{1}{c} \underbrace{\frac{dI_s}{dt}}_{0}$$

① $V_{cp} = 0$

② $V_{ch} = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

③ $V_c = V_{cp} + V_{ch}$
$= A_1 e^{s_1 t} + A_2 e^{s_2 t}$

⟹

④ $V_c(0) = V_0 = A_1 + A_2$

$$\frac{dV_c}{dt}(0) = s_1 A_1 + s_2 A_2$$

$$\frac{dV_c}{dt}(0) = -2\alpha V_0 - \frac{I_0}{C} + \frac{I_s}{C}$$

$$** \quad s_1 A_1 + s_2 A_2 = -\frac{V_0}{RC} - \frac{I_0}{C} + \frac{I_s}{C}$$

Solve "*" and "**" to find A_1 and A_2.

$$V_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad s_1, s_2 \rightarrow \text{real negative (for overdamped)}$$

$$\lim_{t\to\infty} V_c(t) = 0$$

Steady state value for $V_c(t) = 0$

$$\frac{d^2 I_L}{dt^2} + 2\alpha \frac{dI_L}{dt} + \omega_0^2 I_L = \omega_0^2 I_s$$

① $V_{cp} = K$ (input is I_s (DC) so V_{cp} is similar (DC) ($V_{cp} = K$))

$$\underbrace{\frac{d^2}{dt^2} K}_{0} + 2\alpha \underbrace{\frac{d}{dt} K}_{0} + \omega_0^2 K = \omega_0^2 I_s$$

$$\omega_0^2 K = \omega_0^2 I_s$$

$$I_{cp} = K = I_s$$

② $I_{ch} = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

③ $I_L = I_{cp} + I_{ch}$

$$= I_s + K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

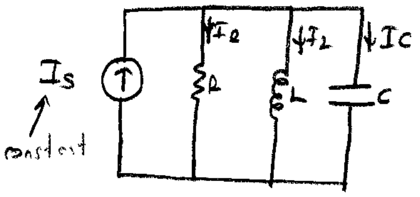
④ $I_L(0) = I_0 = I_s + K_1 + K_2 \quad *$

$$\left.\frac{dI_L}{dt}\right|_{t=0} = s_1 K_1 + s_2 K_2 = \frac{V_0(0)}{L} = \frac{V_0}{L} \quad **$$

$$\left.\begin{array}{l} K_1+K_2+I_s=I_o \\ s_1K_1+s_2K_2=\frac{V_o}{L} \end{array}\right\}\ \text{solve } k_1 \text{ and } k_2$$

$$\lim_{t\to\infty} I_L(t)=\lim_{t\to\infty} I_s+k_1e^{s_1t}+k_2e^{s_2t}=I_s$$

(steady state value for $I_L(t)$ is I_s)

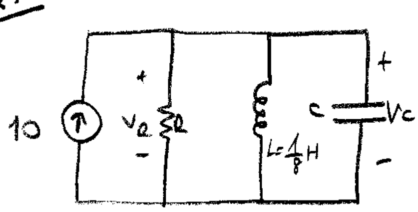


$I_L(\infty)=I_s$

$V_C(\infty)=0$

$t\to\infty$

Ex.



$C=\frac{1}{2}F$ $\quad I_L(0)=-4A$

$V_C(0)=5V$

① $R=\frac{1}{5}$ (overdamped)

$V_c(t)=3e^{-2t}+2e^{-8t}$

$I_L(t)=-12e^{-2t}-2e^{-8t}+10$

$2\alpha=\frac{1}{RC}=\frac{1}{\frac{1}{5}\times\frac{1}{2}}=10$

$w_o^2=\frac{1}{LC}=\frac{1}{\frac{1}{8}\times\frac{1}{2}}=16$

$\alpha=5,\ w_o=4\,rad/sec$

$s^2+2\alpha s+w_o=0$

$s^2+10s+16=0$

$s_{1,2}=-5\pm3$

$s_{1,2}=-2,8$

$\Rightarrow$

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cihanyilmaz@[illegible]

$$\frac{d^2I_L}{dt^2} + 2\alpha \frac{dI_L}{dt} + \omega_0^2 I_L = \omega_0^2 I_s$$

$$\frac{d^2I_L}{dt^2} + 10\frac{dI_L}{dt} + 16 I_L = 16\times 10$$

$$I_L(0) = -4 \qquad \frac{dI_L}{dt}(0) = \frac{V_0}{L} = \frac{5}{1/8} = 40$$

① $I_{Lp} = K$

$$\underbrace{\frac{d^2}{dt^2}K}_{0} + 10\underbrace{\frac{d}{dt}K}_{0} + 16K = 16\times 10$$

$$16K = 16\times 10 \rightarrow K = 10$$

② $I_{ch} = B_1 e^{-2t} + B_2 e^{-8t}$

③ $I_L = I_{cp} + I_{ch}$

$$= 10 + B_1 e^{-2t} + B_2 e^{-8t}$$

④ $I_L(0) = -4 = 10 + B_1 + B_2$

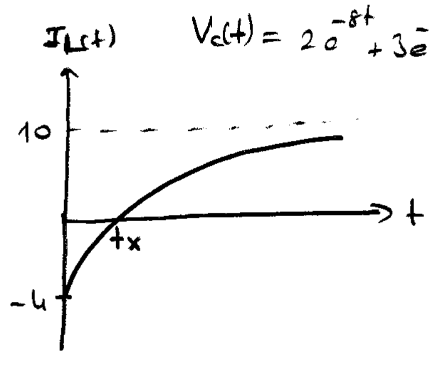
$$\left.\begin{array}{l} \left.\frac{dI_L}{dt}\right|_{t=0} = -2B_1 - 8B_2 = 40 \\ B_1 + B_2 = -14 \end{array}\right\} \begin{array}{l} B_2 = -2 \\ B_1 = -12 \end{array}$$

$$I_L(t) = 10 - 2e^{-8t} - 12e^{-2t}$$

$$L\frac{dI_L}{dt} = V_c = \frac{1}{8}\left[\frac{d}{dt}\left[10 - 2e^{-8t} - 12e^{-2t}\right]\right]$$

$$= \frac{1}{8}\left[16e^{-8t} + 24e^{-2t}\right]$$

$$V_c(t) = 2e^{-8t} + 3e^{-2t}$$



Ramp Response

$I_s(t) = r(t)$

$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + \omega_o^2 V_c = \frac{1}{C}\frac{d}{dt}\overbrace{r(t)}^{I_s(t)}$$

$V_c(0^-) = 0$

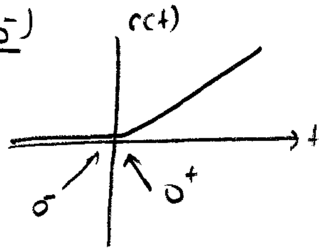
$$\frac{d^2 I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + \omega_o^2 I_L = \omega_o^2\,\overbrace{r(t)}^{I_s(t)}$$

$I_L(0^-) = 0$

$$\frac{dV_c}{dt}(0^-) = -\frac{V_c(0^-)}{RC} - \frac{I_L(0^-)}{C} + \frac{I_s(0^-)}{C}$$

$$= -\frac{0}{RC} - \frac{0}{C} + \frac{\overbrace{r(0^-)}^{0}}{C}$$

$$= 0$$



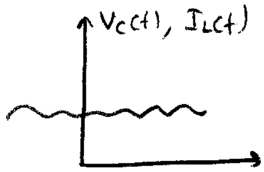
$$\left.\frac{dI_L}{dt}\right|_{t=0^-} = \frac{V_c(0^-)}{L} = 0$$

$t > 0$

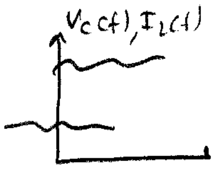
$$\frac{dV_c}{dt} = -\frac{1}{RC}V_c - \frac{1}{C}I_L + \frac{I_s}{C}$$

$$\int_{V_c(0^-)}^{V_c(0^+)} dV_c = -\frac{1}{RC}\int_{0^-}^{0^+} V_c(t')dt' - \frac{1}{C}\int_{0^-}^{0^+} I_L(t')dt' + \frac{1}{C}\int_{0^-}^{0^+} \overbrace{I_s(t')}^{r(t')}dt'$$

$V_c(0^+) - V_c(0^-) = 0 - 0 + 0 \rightarrow V_c(0^+) = V_c(0^-)$ ✓



$\int_{0^-}^{0^+} V_c(t')dt' = 0$



$\int_{0^-}^{0^+} V_c(t')dt' = 0$

$$\frac{dI_L}{dt} = \frac{V_c}{L}$$

$$\int_{I_L(0^-)}^{I_L(0^+)} I_L = \frac{1}{L}\int_{0^-}^{0^+} V_c(t')dt'$$

$I_L(0^+) - I_L(0^-) = 0$ ✓

$I_L(0^+) = I_L(0^-) = 0$ ✓

$$\frac{dI_L}{dt}(0^+) = \frac{V_c(0^+)}{L} = \frac{0}{L} = 0 \checkmark$$

$$\frac{dV_c}{dt}(0^+) = -\frac{1}{RC}V_c(0^+) - \frac{1}{C}I_L(0^+) + \frac{I_s(0^+)}{C}$$

$$= 0 - 0 + \frac{r(0^+)}{C}$$

$$= 0$$

$t > 0$

$$\frac{d^2V_c}{dt^2} + \underbrace{\frac{1}{RC}}_{2\alpha}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{1}{C}\,1$$

$V_c(0^+) = 0 \quad \frac{dV_c}{dt}(0^+) = 0$

$$\frac{d^2I_L}{dt^2} + \frac{1}{RC}\frac{dI_L}{dt} + \frac{1}{LC}I_L = \frac{1}{LC}\,t \quad r(t)$$

$I_L(0^+) = 0 \quad \frac{dI_L}{dt}(0^+) = 0$

<u>assume system overdamped;</u>

Char eq $\Rightarrow s^2 + \underbrace{\frac{1}{RC}}_{2\alpha}s + \underbrace{\frac{1}{LC}}_{\omega_0^2} = 0$

$S_{1,2} \rightarrow$ real negative

For I_L

① $I_{Lp} = A_1 t + A_2$

$$\underbrace{\frac{d^2}{dt^2}(A_1t + A_2)}_{0} + \underbrace{\frac{1}{RC}\frac{d}{dt}(A_1t + A_2) + \frac{1}{LC}(A_1 + A_2)}_{} = \frac{1}{LC}t$$

$$\frac{1}{RC}A_1 + \frac{1}{LC}(A_1t + A_2) = \frac{1}{LC}t$$

$\Rightarrow$

$$\left(\underbrace{\frac{1}{RC}A_1 + \frac{1}{LC}A_2}_{0}\right)t + \left(\underbrace{\frac{A_1}{LC} - \frac{1}{LC}}_{0}\right) = 0$$

$$A_1 = 1$$

$$\frac{1}{RC}A_1 = -\frac{1}{LC}A_2$$

$$A_2 = -\frac{L}{R} \,//$$

(2) $I_{Lh} = B_1 e^{s_1 t} + B_2 e^{s_2 t}$

(3) $I_L = I_{Lh} + I_{cp}$

$$= B_1 e^{s_1 t} + B_2 e^{s_2 t} + t - \frac{L}{R}$$

$$\left.\begin{aligned} I_L(0^+) = 0 &= B_1 + B_2 - \frac{L}{R} \\ \frac{dI_L}{dt}(0^+) &= s_1 B_1 + s_2 B_2 + 1 = 0 \end{aligned}\right\}$$ solve for B_1 and B_2

$$B_1 = -\frac{1}{s_1 + s_2}\left(1 + \frac{s_2 L}{R}\right) \qquad B_2 = \frac{1}{s_1 - s_2}\left(1 + \frac{s_1 L}{R}\right)$$

$$\lim_{t\to\infty} I_L(t) \cong t - \frac{L}{R}$$ (at steady state)

$$\lim_{t\to\infty} I_L(t) = \infty$$

$$L\frac{dI_L}{dt} = V_C = L\left[\frac{d}{dt}\left[B_1 e^{s_1 t} + B_2 e^{s_2 t} + t - \frac{L}{R}\right]\right]$$

$$V_C(t) = L\left[s_1 B_1 e^{s_1 t} + s_2 B_2 e^{s_2 t} + 1\right]$$

$$V_C(t) = \underbrace{\overbrace{s_1 L B_1}^{A_1} e^{s_1 t} + \overbrace{s_2 L B_2}^{A_2} e^{s_2 t}}_{V_{ch}} + \underbrace{L}_{V_{cp}}$$

$$A_1 = -\frac{s_1 L}{s_1 - s_2}\left(1 + \frac{s_2 L}{R}\right) \qquad A_2 = \frac{s_2 L}{s_1 - s_2}\left(1 + \frac{s_1 L}{R}\right)$$