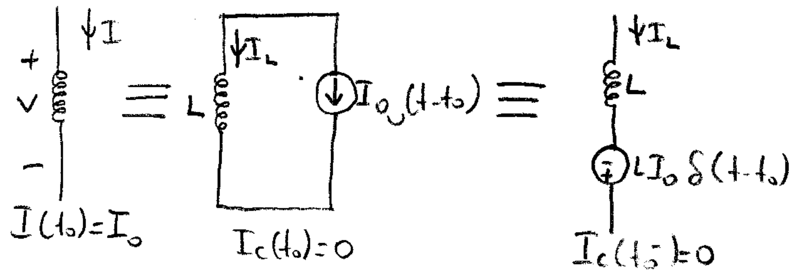
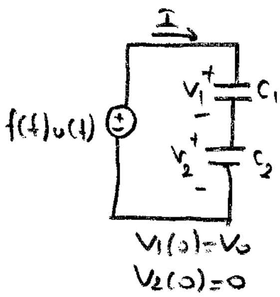
For Inductor

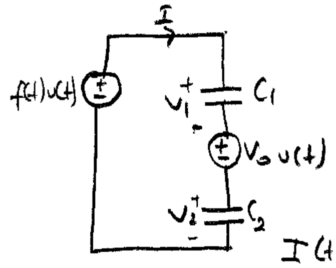


Voltage Division



Find $V_1(t)$, $V_2(t)$

Using initial condition models



$$I(t) = C_{Total}\frac{d}{dt}V_{total}$$

$$I(t) = \frac{C_1C_2}{C_1+C_2}\frac{d}{dt}\left[f(t)u(t) - V_0 u(t)\right]$$

$$I(t) = \frac{C_1C_2}{C_1+C_2}\left[f'(t)u(t) + \underbrace{f(t)\delta(t)}_{f(0)\delta(t)} - V_0\delta(t)\right]$$

$$I(t) = \frac{C_1C_2}{C_1+C_2}\left[f'(t)u(t) + (f(0)-V_0)\delta(t)\right]$$

$$V_1(t) = V_1(0) + \frac{1}{C_1}\int_0^t I(t')dt'$$

$$V_1(t) = V_1(0) + \frac{1}{C_1}\int_0^t \frac{C_1C_2}{C_1+C_2}\left[f'(t')u(t') + (f(0)-V_0)\delta(t')\right]dt'$$

$$V_1(t) = V_1(0) + \frac{C_2}{C_1+C_2}\left[\int_0^t f'(t')u(t')dt' + \int_0^t (f(0)-V_0)\delta(t')dt'\right]$$

$$= V_1(0) + \frac{C_2}{C_1+C_2}\left[f(t)-f(0)\right]u(t) + \frac{C_1}{C_1+C_2}\left[f(0)-V_0\right]u(t)$$

$$= V_1(0) + \frac{C_2}{C_1+C_2}\left[f(t)-V_0\right]u(t)$$

$$= V_0 + \frac{C_2}{C_1+C_2}\left[f(t)-V_0\right]u(t) \rightarrow \boxed{V_0 + \frac{C_2}{C_1+C_2}(f(t)-V_0) \text{ when } t>0}$$

For $V_2(t)$

$$V_2(t) = f(t)u(t) - V_1(t)$$

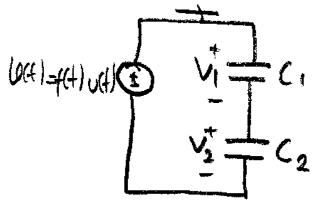
$$= f(t)u(t) - \left[V_0 + \frac{C_2}{C_1+C_2}\left[f(t)-V_0\right]u(t)\right]$$

$$= f(t)u(t) - V_0 - \frac{C_2}{C_1+C_2} f(t)u(t) + V_0 \frac{C_2}{C_1+C_2} u(t)$$

$$= f(t)u(t)\left(\frac{C_1}{C_1+C_2}\right) + V_0\frac{C_2}{C_1+C_2}u(t) - V_0 \quad \text{for } t>0$$

$$\cong \left[f(t)-V_0\right]\frac{C_1}{C_1+C_2}u(t)$$

Voltage Division 2

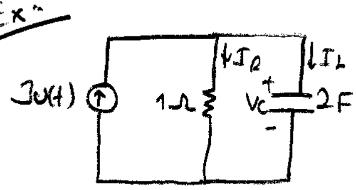


$$V_1(t) = \frac{C_2}{C_1+C_2} i(t)$$

$$V_2(t) = \frac{C_1}{C_1+C_2} i(t)$$

$v_1(0) = V_2(0) = 0$

Ex:



a) $V_C(0^-)=0$, find $V_C(0^+), I_R(0^-), I_R(0^+)$
$I_C(0^-), I_C(0^+)$

* Impulsive sources might occur when;
  - There is really an impulsive current or voltage source.
  - Connection of an extra circuit element (like capacitor, inductor) is provided.

$V_C(0^+) = V_C(0^-) = 0$ (Continous since no impulsive source exists)

$$I_R(0^-) = \frac{V_R(0^-)}{R} = \frac{V_C(0^-)}{R} = \frac{0}{R} = 0$$

$$I_s = 3u(t) = I_R + I_L$$

$$I_S(0^-) = I_R(0^-) + I_L(0^-)$$

$$3u(0^-) = 0 + I_L(0^-) \rightarrow I_L(0^-) = 0$$

$$0 = 0 \rightarrow I_L(0)$$

$$I_R(0^+) = \frac{V_R(0^+)}{R} = \frac{V_C(0^+)}{R} = \frac{0}{R} = 0$$

$$I_S(0^+) = I_R(0^+) + I_C(0^+)$$

$$3u(0^+) = 0 + I_C(0^+)$$

$$3 = 0 + I_C(0^+)$$

$$I_C(0^+) = 3$$

b) If $V_C(0^-) = 4\text{ Volt}$

$$V_C(0^+) = V_C(0^-) = 4\text{ Volt}$$

$$I_R(0^-) = \frac{V_C(0^-)}{R} = \frac{4}{1} = 4\text{ Amper}$$

$$I_S(0^-) = I_R(0^-) + I_C(0^-)$$

$$3u(0^-) = 4 + I_C(0^-)$$

$$0 = 4 + I_C(0^-)$$

$$I_C(0^-) = -4\text{ Amper}$$

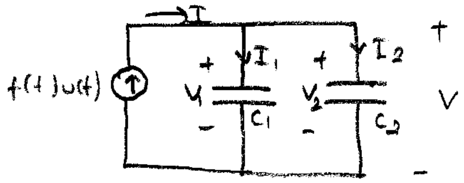
$$I_R(0^+) = \frac{V_C(0)}{R} = \frac{4}{1} = 4\text{ Amper}$$

$$I_S(0^+) = I_R(0^+) + I_C(0^+)$$

$$3 = 4 + I_C(0^+)$$

$$I_C(0^+) = -1\text{ Amper}$$

Current Division



$$V_1(0^-) = V_2(0^-) = 0$$

$$I_1 = C_1 \frac{dV_1}{dt} \qquad I_2 = C_2 \frac{dV_2}{dt} \qquad C_{Total} = C_1 + C_2$$

$$V(t) = V(0) + \frac{1}{C_{total}} \int_0^t I(t')dt'$$

$$V(t) = V(0) + \frac{1}{C_1+C_2} \int_0^t f(t')u(t')dt'$$

$$\int f(t)dt = F(t)$$

$$V(t) = 0 + \frac{1}{C_1+C_2}\left[F(t) - F(0)\right]u(t)$$

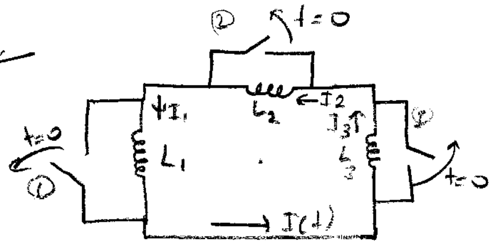
$$I_1 = C_1 \frac{dV_1}{dt} = C_1 \frac{dV}{dt}$$

$$= C_1 \frac{d}{dt} \frac{1}{C_1+C_2}\left[F(t)u(t) - F(0)u(t)\right]$$

$$= \frac{C_1}{C_1+C_2}\left[f(t)u(t) + F(t)\delta(t) - F(0)\delta(t)\right]$$

$$I_1(t) = \frac{C_1}{C_1+C_2} f(t)u(t)$$

Ex.



When t=0 all the three switches ①, ②, ③ are opened

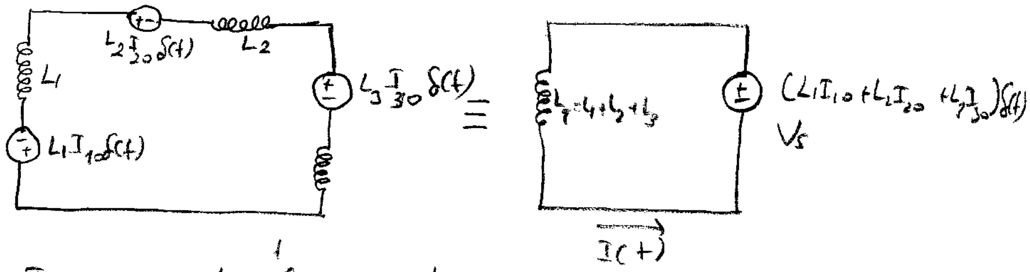
$I_1(0^-) = I_{10}$

$I_2(0^-) = I_{20}$

$I_3(0^-) = I_{30}$

Find $I(t)$ $t > 0$

Use initial condition models:

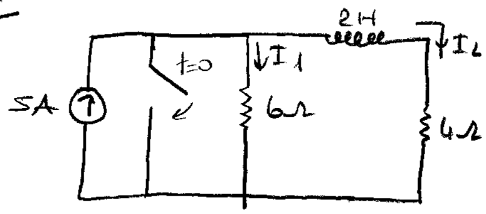


$$I(t) = I(0) + \frac{1}{L_{total}} \int_0^t V_s(t')\,dt'$$

$$I(t) = 0 + \frac{1}{L_1 + L_2 + L_3} \int_0^t (L_1 I_{10} + L_2 I_{20} + L_3 I_{30})\,\delta(t')\,dt'$$

$$\boxed{I(t) = \frac{(L_1 I_{10} + L_2 I_{20} + L_3 I_{30})\,u(t)}{L_1 + L_2 + L_3}}$$

Ex.

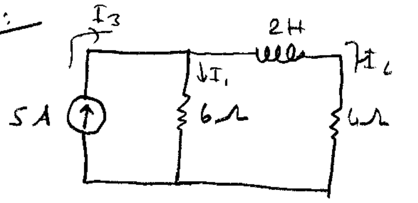


$I_1(0^-) = 2A$

Find $I_L(0^-)$, $I_L(0^+)$, $I_1(0^+)$

on $\frac{dI}{dt}(0^+)$ if the switch is closed when $t=0$

Soln:



$I_3 = I_1 + I_L$

$I_s(0^-) = I_1(0^-) + I_L(0^-)$

$5 = 2 + I_L(0^-)$

$I_L(0^-) = 3A$

$I_L(0^+) = I_2(0^-) = 3A$ (due to continuity property)

$V_1(0^+) = 0$ (since short-circuit exist)

$$I_1(0^+) = \frac{V_1(0^+)}{6} = 0$$

$$\frac{dI_L}{dt}(0^+) = ?$$

$$V_1 = V_L + V_2$$

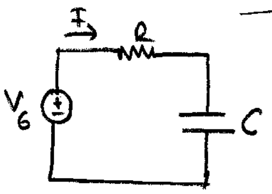
$$V_1(0^+) = V_L(0^+) + V_2(0^+)$$

$$0 = L\frac{dI_L(0^+)}{dt} + I_L(0^+).4 - 4I_L(0^+) = L\frac{dI_L}{dt}(0^+) - 2I_L(0^+) = \frac{dI}{dt}(0^+) = -6$$

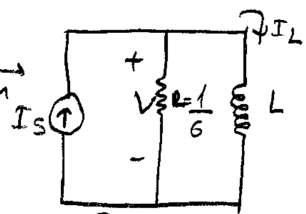
First Order Circuits

C + R ↘ First order circuits
L + R ↗

RC and RL Circuits



$\xrightarrow[\text{problem}]{\text{dual}}$



$V_C(0) = V_0$

$V_C \rightarrow$ State Variable

$I_L(0) = I_0$

$I_L \rightarrow$ state variable

State variables determine the energy stored in the system.

$$I(t) = C \frac{dV_C}{dt}$$

$$-V_S + \underbrace{V_R}_{RI} + V_C = 0$$

$$RC \frac{dV_C}{dt} + V_C = V_S$$

$$\boxed{\frac{dV_C}{dt^2} + \frac{1}{RC} V_L = \frac{1}{RC} V_S}$$

$$V_L(0) = V_0$$

state equations →

$$V(t) = L \frac{dI_L}{dt}$$

$$I_S = \underbrace{I_R}_{\frac{V}{R}} + I_L$$

$$I_S = \frac{L}{R} \frac{dI_L}{dt} + I_L$$

$$\boxed{\frac{dI_L}{dt} + \frac{1}{LG} I_L = \frac{1}{LG} I_S}$$

$$I_L(0) = I_0$$

First order linear differential equations with constant coefficients

The response V_c and I_L depends on;

① The inputs "V_s", "I_s"

② Circuit parameters ; R, C, L

③ Initial Conditions V_0, I_0

The initially stored energy in the circuit due to I_0 and V_0 will produce a non-zero solution ($V_c(t) \neq 0, I_c \neq 0$) even if there is no input to the system. ($V_0(t)=0$ and $I_c(t)=0$) (Known as zero-input response)