30.06.2010

Superposition Theorem

For LTI resistive circuit, if the circuit includes more than one independent source, the response associated with any variable over the circuit can be calculated as follow (variable any current or voltage value over any circuit element)

Step 1- kill sources 1.... n-1, find the effect of n^{th} source over the variable: Eff_n

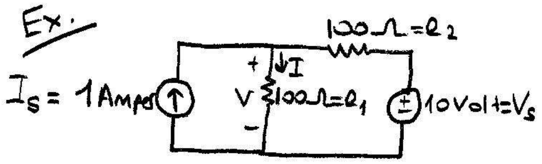
Step 2- kill sources 1, ..., n-2, n, find the effect of $n-1^{th}$ source over the variable; Eff_{n-1}

⋮

Step n - kill sources 2, ..., n find " " " 1^{st} " " the variable; Eff_1

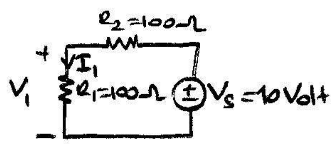
$$Eff_{Total} = \text{Algebric sum}\,(Eff_1, \dots Eff_n)$$

Ex.



Use superposition to find V and I.

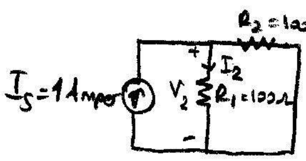
Sltn: Step1→ Kill Is (open-circuit the branch) and see effect of Vs.



$$V_1 = V_s \frac{R_1}{R_1+R_2} = 10\,\frac{100}{100+100} = 5\,V$$

$$I_1 = \frac{V_s}{R_1+R_2} = \frac{10}{100+100} = 0{,}05 \text{ Amper}$$

step 2 → Kill Vs (Short Circuit the branch), see effect of Is



$$I_2 = I_s \frac{R_2}{R_1+R_2} = 1\,\frac{100}{100+100} = 0.5 \text{ Amper}$$

$$V_2 = R_1 I_2 = 100 \times 0.5 = 50 \text{ Volt}$$

Finally ⟶ $V = V_1 + V_2$

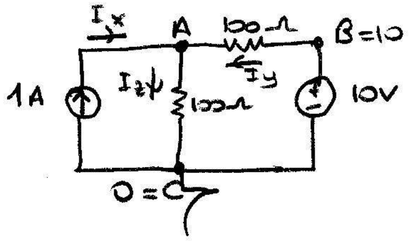
$= 5 + 50 = 55$ Volt

$I = I_1 + I_2$

$= 0.05 + 0.5 = 0.55$ Amper

⟹ Continues

Using Node Voltage without Killing the Sources:



$I_x + I_y = I_z$

$I + \frac{B-A}{100} = \frac{A-C}{100}$

$100 + B - A = A - C$

$100 + B + C = 2A$

$110 = 2A$

$A = 55$ Volt

$V = A - C$

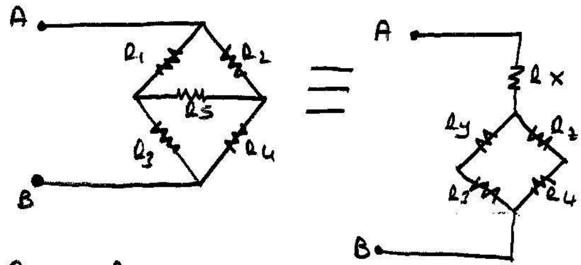
$= 55$ Volt

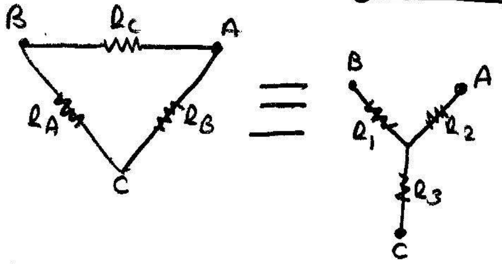
$I_z = \frac{A-C}{100}$

$= \frac{55}{100} = 0.55\,A$

Same Result!

Δ (Delta) – Y Connection



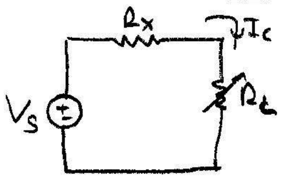


$R_1 = \frac{R_A R_C}{R_A + R_B + R_C}$ $\quad R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$

$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$ $\quad R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$

$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$ $\quad R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$

Maximum Power Transfer



What is the value for R_L such that $P_L = I_L^2 R_L$ is maximum.

$I_L = \frac{V_S}{R_x + R_L}$

$P_L = \left(\frac{V_S}{R_x + R_L}\right)^2 R_L = \frac{V_S^2}{(R_x + R_L)^2} R_L$

To find the R_L value where P_L is maximized.

$$\frac{dP_L}{dR_L} = 0 = V_s^2 \left[\frac{(R_x+R_L)^2 - R_L(2)(R_x+R_L)}{(R_x+R_L)^4} \right] = 0$$

$$(R_x+R_L)^2 - 2R_L(R_x+R_L) = 0$$

$$(R_x+R_L)\left[R_x+R_L-2R_L\right] = 0$$

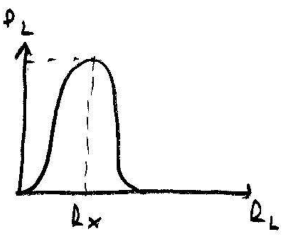
$$(R_x+R_L)(R_x-R_L) = 0$$

$R_x+R_L = 0 \quad R_x = -R_L$ (not valid since $R_L>0$)

$R_x-R_L = 0 \quad R_x = R_L$ (valid)

For maximum power trasfer to R_L, $R_L = R_x$

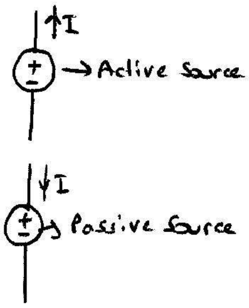
$$P_{L_{max}}\Big|_{R_L=R_x} = \frac{V_s^2 R_x}{(R_x+R_x)^2} = \frac{V_s^2}{4R_x} \qquad P_{L_{max}} = \frac{V_s^2}{4R_x}$$

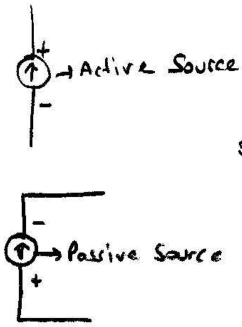


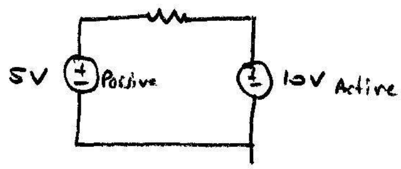
Ex.

Active source → It gives power to it's circuit

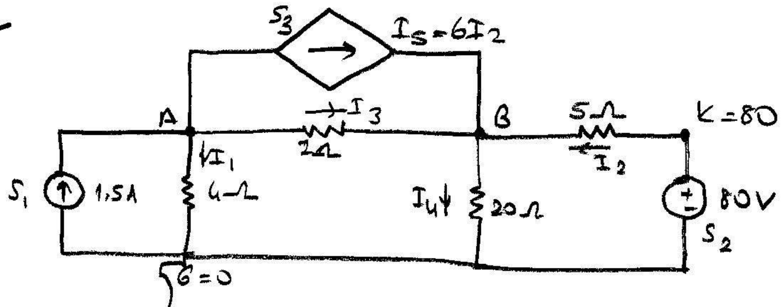
Passive source → It takes power from the circuit.







Ex.



Find the power associated with the sources indicating whether they are active or passive.

Soltn.

Node A

$$1.5 = I_1 + I_3 + \underbrace{I_S}_{6I_2} \qquad \boxed{1.5 = \frac{A-G}{4} + \frac{A-B}{2} + 6\,\frac{K-B}{5}} \qquad K=80,\ G=0$$

Node B

$$I_3 + I_2 + \underbrace{I_S}_{6I_2} = I_4 \qquad \boxed{\frac{A-B}{2} + 7\,\frac{K-B}{5} = \frac{B-G}{20}} \qquad K=80,\ G=0$$

$$I_3 + 7I_2 = I_4$$

A = 10 Volt
B = 60 Volt

For S1

$V_1 = A - G = 10 - 0 = 10V$ S_1 is active source

For S2

$I_2 = \frac{80-60}{5} = 4A$ S_2 is active source

For S3

$I_2 = 4A \rightarrow I_S = 24A$ $(B-A) = V_3 = 50$ S_3 is active source

Total Power Supplied to the Circuit

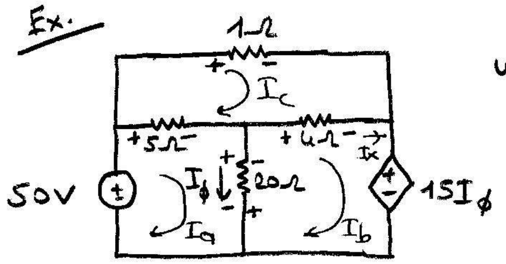
$$P_T = P_{S_1} + P_{S_2} + P_{S_3}$$ (since all sources are active)

$$P_T = V_1 \cdot 1.5 + 80 I_2 + V_3 6 I_2$$

$$= 10 \times 1.5 + 80 \times 4 + 50 \times 24$$

$$= 1535 \text{ watt}$$

Ex.



what is $P_{4\Omega}$?

Soltn.

Mesh I_a

$$-50 + 5(I_a - I_c) + 20(I_a - I_b) = 0$$

Mesh I_c

$$1I_c + 4(I_c - I_b) + 5(I_c - I_a) = 0$$

Mesh I_b

$$20(I_b - I_a) + 4(I_b - I_c) + 15\underbrace{I_\phi}_{(I_a - I_b)} = 0$$

$$50 = 25I_a - 20I_b - 5I_c$$

$$0 = -5I_a + 9I_b - 4I_c$$

$$0 = -5I_a - 4I_b + 10I_c$$

$$\begin{bmatrix} 25 & -20 & -5 \\ -5 & 9 & -4 \\ -5 & -4 & 10 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$I_b = 28A$

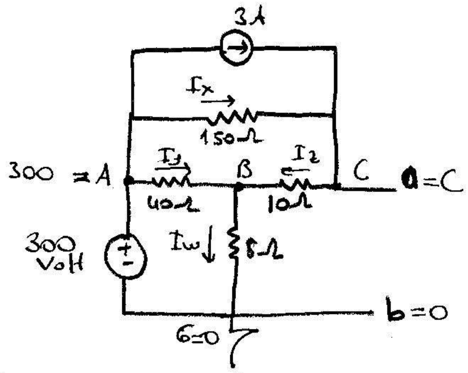
$I_c = 26A$

$$I_x = I_b - I_c = 28 - 26 = 2A$$

$$P_{4\Omega} = 4 \times (2)^2 = 16 \text{ Watt.}$$

Ex.

Find thevenin equivalent circuit w.r.t. a-b



Soltn.

For $V_{Th} = V_{ab}$

<u>Node B</u>

$I_y + I_2 = I_w$

$$\boxed{\frac{A-B}{40} + \frac{C-B}{10} = \frac{B-0}{8}}$$

<u>Node C</u>

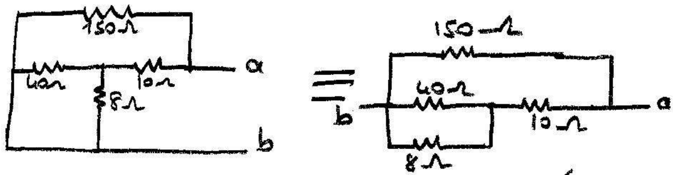
$3 + I_x = I_2$

$$\boxed{3 + \frac{A-C}{150} = \frac{C-B}{10}}$$

$C = 120$, $V_{ab} = C - b = 120 - 0$

$$\boxed{V_{Th} = V_{ab} = 120}$$

For R_{Th}, kill independent sources;



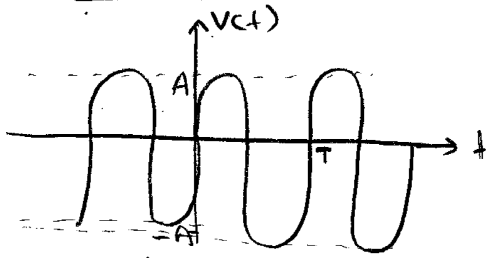
$R_{ab} = R_{Th} = \left((40 // 8) + 10\right) // 150$

$R_{Th} = \left(\frac{40 \times 8}{40+8} + 10\right) // 150$

$= \left(\frac{800}{48}\right) // 150 = 15\,\Omega$

Basic Concepts About AC Analysis

Sine Wave



T = Period (seconds)

$f = \frac{1}{T}$ (Hz)

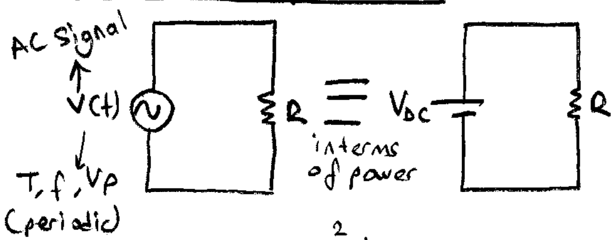
$V(t) = V_p \sin(2\pi f t)$

$V_p = A \rightarrow$ peak value

$-V_p = -A$

$V_{pp} = 2A \rightarrow$ peak-to-peak value

Root-mean-square value (RMS)



* what is the value for V_{DC} such that (V_{DC} is a DC Value) both circuits produce same amount of energy in the same period of time?

$$P(t) = \frac{V^2(t)}{R}$$

Instantaneous Power

$$E(t) = \int_0^t P(t')\,dt' = \int_0^t \frac{V^2(t')}{R}\,dt'$$

$$E(T) = \int_0^T \frac{V^2(t')}{R}\,dt'$$

energy produced in a single period for AC circuit

For the DC circuit

$$P = \frac{V_{DC}^2}{R}$$

$$E(T) = \frac{V_{DC}^2}{R}\,T$$

$$\int_0^T \frac{V^2(t')}{R}\,dt' = \frac{V_{DC}^2}{R}\,T$$

$$V_{DC} = \sqrt{\frac{1}{T}\int_0^T V^2(t')\,dt'}$$

$V_{RMS} \rightarrow V_{\text{Root-Mean-Square}}$

Let's find RMS of a sinewave;

$$V(t) = V_p \sin(2\pi f t) = V_p \sin\left(\frac{2\pi t}{T}\right)$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_p^2 \sin^2\left(\frac{2\pi t}{T}\right) dt'}$$

$$= \sqrt{\frac{1}{T}\int_0^T V_p^2 \left(\frac{1-\cos\left(\frac{4\pi t}{T}\right)}{2}\right) dt'}$$

$$= \sqrt{\frac{V_p^2}{2T}\left(t' - \frac{T}{4\pi}\sin\left(\frac{4\pi t'}{T}\right)\right)\Big|_0^T}$$

$$= \sqrt{\frac{V_p^2}{2T}\left(T - \frac{T}{4\pi}\sin\left(\frac{4\pi T}{T}\right) - \left(0 - \frac{1}{T\pi}\sin(0)\right)\right)}$$

$$= \sqrt{\frac{V_p^2 \cancel{T}}{2\cancel{T}}} = \frac{V_p}{\sqrt{2}}$$

$$\boxed{\text{for sine wave } V(t) = V_p \sin\left(\frac{2\pi t}{T}\right) \longrightarrow V_{RMS} = \frac{V_p}{\sqrt{2}}}$$

Average Value

$$V_{avg} = \frac{1}{T}\int_0^T V(t')\,dt'$$

V_{avg} gives the DC content in a signal

in oscilloscope; V_{avg} formula

DC pressed $\longrightarrow$ DC + AC

AC pressed $\longrightarrow$ AC

Let's calculate average value for a sinewave;

$$V(t) = V_p \sin\left(\frac{2\pi t}{T}\right)$$

$$V_{avg_{sine}} = \frac{1}{T}\int_0^T V_p \sin\left(\frac{2\pi t}{T}\right) dT$$

$$= \frac{1}{T} V_p \frac{T}{2\pi}\left(-\cos\left(\frac{2\pi t}{T}\right)\right)\Big|_0^T$$

$$= \frac{-V_p}{2\pi}\left[\left(\underbrace{\cos(2\pi)}_{1} - \underbrace{\cos(0)}_{1}\right)\right]$$

$$V_{avg_{sine}} = 0$$

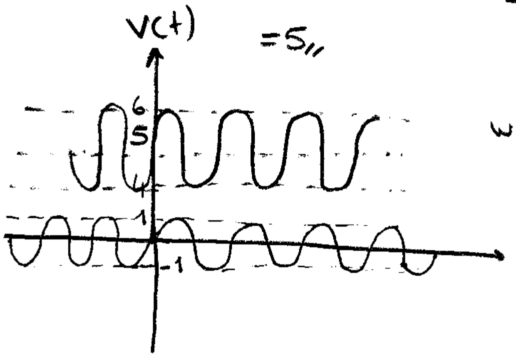
Ex. $V(t) = 5t \sin\left(\frac{2\pi t}{T}\right)$ What is $V_{avg_{v(t)}}$?

$$V_{avg_{v(t)}} = \frac{1}{T}\int_0^T \left[5t\sin\left(\frac{2\pi t}{T}\right)\right]dt$$

$$= \frac{1}{T}\left[5t - \frac{T}{2\pi}\cos\left(\frac{2\pi t}{T}\right)\right]\Big|_0^T$$

$$= \frac{1}{T}\left[5(T-0) - \frac{T}{2\pi}\left(\cos(2\pi) - \cos(0)\right)\right]$$

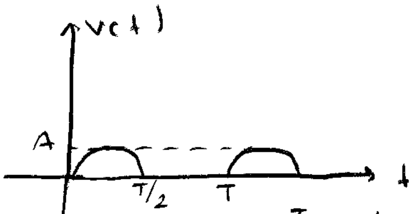
$$= 5$$



when DC is pressed

when AC is pressed

Ex.



$$V(t) = \begin{cases} A\sin\left(\frac{2\pi t}{T}\right) & \text{when } 0 < t < T/2 \\ 0 & \text{when } T/2 < t < T \end{cases}$$

$V(t)$ periodic with T. V_{avg} and V_{rms}?

$$V_{avg_{v(t)}} = \frac{1}{T}\int_0^T v(t')\,dt'$$

$$= \frac{1}{T}\left[\int_0^{T/2} A\sin\left(\frac{2\pi t}{T}\right)dt' + \int_{T/2}^T 0\,dt\right]$$

$$= \frac{1}{T}\left[A\frac{T}{2\pi}\left[-\cos\left(\frac{2\pi t}{T}\right)\right]\Big|_0^{T/2} + 0\right]$$

$$= \frac{1}{T}\left[\frac{-AT}{2\pi}\left[\cos(\pi) - \cos(0)\right] + 0\right]$$

$$= -\frac{A}{T}\frac{(-2)T}{2\pi} = \frac{A}{\pi}$$

$$V_{RMS_{V(t)}} = \sqrt{\frac{1}{T}\int_0^T V^2(t')\,dt'}$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{T/2} A^2 \sin^2\left(\frac{2\pi t'}{T}\right)dt' + \int_{T/2}^{T} 0^2\,dt'\right]}$$

$$= \sqrt{\frac{A^2}{T}\;\frac{1}{2}\int_0^{T/2}\left(1-\cos\left(\frac{4\pi t'}{T}\right)\right)dt'}$$

$$= \sqrt{\frac{A^2}{T}\;\frac{1}{2}\left[t' - \frac{T}{4\pi}\sin\left(\frac{4\pi t'}{T}\right)\right]\Bigg|_0^{T/2}}$$

$$= \sqrt{\frac{A^2}{T}\;\frac{1}{2}\left[\left(\frac{T}{2}-0\right) - \underbrace{\frac{T}{4\pi}\left(\sin(2\pi) - \sin(0)\right)}_{0}\right]}$$

$$= \sqrt{\frac{A^2}{T}\cdot\frac{1}{2}\cdot\frac{T}{2}} = \frac{A}{2} \quad // \qquad V_{RMS_{V(t)}} = \frac{A}{2} \quad //$$