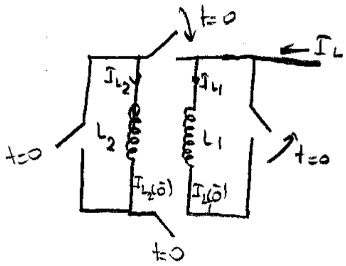
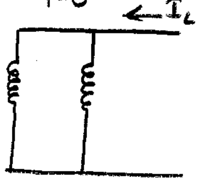
27.07.2010



$t=0^+$



$\cancel{I_{L_2}(0^+) = I_{L_2}(0^-)}$

$\cancel{I_{L_1}(0^+) = I_{L_1}(0^-)}$

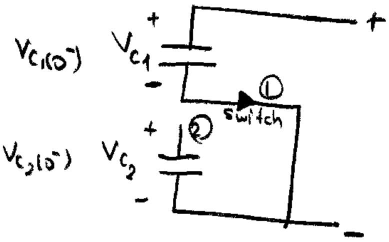
$L_{Total} = \frac{L_1 L_2}{L_1 + L_2}$

$E_{initial} > E_{final}$

$$\frac{1}{2} L_1 I_{L_1}^2(0^-) + \frac{1}{2} L_2 I_{L_2}^2(0^-) \geq \frac{1}{2}\left(\frac{L_1 L_2}{L_1+L_2}\right)\left[I_{L_1}(0^+) + I_{L_2}(0^+)\right]^2$$

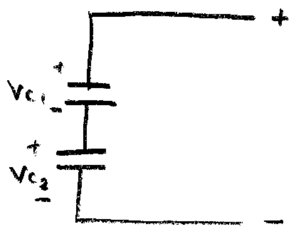
$$\boxed{I_L(0^+) = I_{L_1}(0^+) + I_{L_2}(0^+)}$$

if I_{L_2} is reversed $\Rightarrow \boxed{I_L(0^+) = I_{L_1}(0^+) - I_{L_2}(0^+)}$



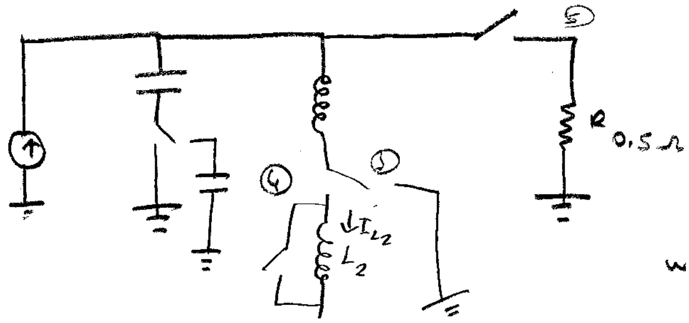
at t=0 switch is brought to position 2 from position 1

when $t=0^+$



$E_{initial} \geq E_{final}$

eski final sorusu 2008-2009



0<t<10π
$S_1 \to 1$
$S_2 \to 3$
$S_3 \to$ closed
$S_4 \to$ "

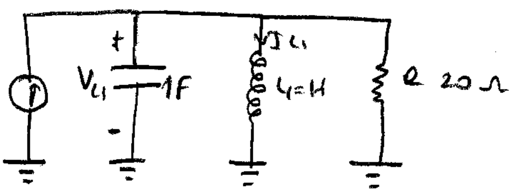
when t>10π
$S_1 \to 2$
$S_2 \to 4$
S_3, S_4) open

Find $I_{L_1}(t) > 0$

Hint = w 0<t<10π

$I_{L_p} = K t^2 e^{-t}$ where I_{L_p} is the particular solution for I_{L_1}

$C_1 = C_2 = 1F$
$L_1 = L_2 = 1H$
$V_{C_1}(0) = 0V$
$V_{C_2}(0) = 2V$
$I_{L_2}(0) = 2A$
$I_{L_1}(0) = 0A$
$I_S(t) = e^{-t}$



$V_{C_2}(10\pi^-) = V_{C_2}(0)$

$I_{L_2}(10\pi^-) = I_{L_2}(0)$

$$\frac{d^2V_C}{dt^2} + \frac{1}{RC_1}\frac{dV_C}{dt} + \frac{1}{L_1C_1}V_{C_1} = \frac{1}{C_1}\frac{dI_S}{dt}$$

$$\frac{d^2I_L}{dt^2} + \frac{1}{RC_1}\frac{dI_{L_1}}{dt} + \frac{1}{L_1C_1}I_{L_1} = \frac{1}{L_1C_1}I_S(t)$$

$V_{C_1}(0) = 0$

$I_L(0) = 0$

$$\frac{I_{L_1}(0)}{dt} = \frac{V_{C_1}(0)}{L} = 0$$

$$\frac{d^2I_{L_1}}{dt^2} + 2\frac{dI_{L_1}}{dt} + I_{L_1} = e^{-t}$$ put I_{L_p} in diff-eqn.

$$\frac{d^2}{dt^2}(kt^2e^{-t}) + 2\frac{d}{dt}(kt^2e^{-t}) + (kt^2e^{-t}) = e^{-t}$$

$$k[2(e^{-t}-te^{-t}) - (2te^{-t}-t^2e^{-t})] + 2k[2te^{-t}-t^2e^{-t}] + kt^2e^{-t} = 1e^{-t}$$

$$k = \frac{1}{2}$$

$$I_{L1p} = \frac{1}{2}t^2e^{-t} \quad (0<t<10\pi)$$

② char eqn.

$s^2+2s+1=0$ critically damped

$s_{1,2} = -1$

$I_{L1n} = A_1e^{-t} + A_2te^{-t}$

we know that $I_s(t) = e^{-t}$, so $I_{L1p} = kt^2e^{-t}$

③ $I_{L1} = I_{L1p} + I_{L1h} = \frac{1}{2}t^2e^{-t} + A_1e^{-t} + A_2te^{-t}$

for A_1

$I_{L1}(0) = 0 = A_1 \qquad \frac{dI_{L1}(0)}{dt} = 0 = \frac{V_{L1}(0)}{L}$

$$\frac{d}{dt}\left(\frac{1}{2}t^2e^{-t} + A_2te^{-t}\right)\Big|_{t=0} = 0$$

$A_2 = 0$

when $0<t<10$

$I_{L1}(t) = \frac{1}{2}t^2e^{-t}$

$V_{L1} = V_{C1} = L\frac{dI_{L1}}{dt} = 1\left[\frac{d}{dt}\left[\frac{1}{2}t^2e^{-t}\right]\right]$

$V_{C1} = \frac{1}{2}\left[2te^{-t} - t^2e^{-t}\right]$

$\Rightarrow$

$\underline{0 < t < 10\pi}$

$I_{L_1} = \frac{1}{2} t^2 e^{-t}$ $V_{C_1} = \frac{1}{2}\left[2t e^{-t} - t^2 e^{-t}\right]$

$I_{L_1}(10\pi^-) = \frac{1}{2}(10\pi)^2 e^{-10\pi} \approx 0$

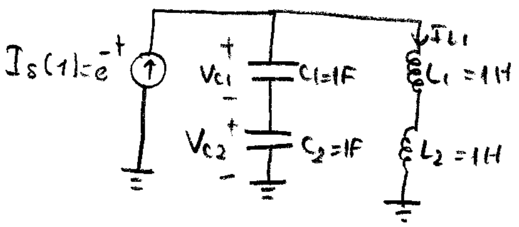
$V_{C_1}(10\pi^-) = \frac{1}{2}\left[2(10\pi) e^{-10\pi} - (10\pi)^2 e^{-10\pi}\right] \approx 0$

$\boxed{e^{-x} \approx 0 \text{ when } x > 5 \text{ (approximation)}}$

$I_{L_1}(10\pi^-) \cong 0$

$V_{C_1}(10\pi^-) \cong 0$

<u>when $t = 10\pi$</u>



$V_C(10\pi^+) = V_{C_1}(10\pi^-) + V_{C_2}(10\pi^-)$

$0 + 1$

$V_C(10\pi^+) = 1$

$\text{flux}(10\pi^-) = \text{flux}(10\pi^+)$

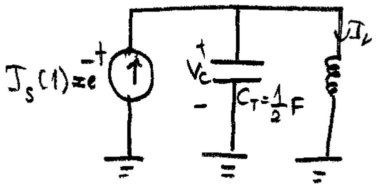
$L_1 I_{L_1}(10\pi^-) + L_2 I_{L_2}(10\pi^-) = (L_1 + L_2) I_L(10\pi^+)$

$1 \times 0 + 1 \times 2 = (1+1) I_L(10\pi^+)$

$I_L(10\pi^+) = 1$ Ampers

$C_T = \frac{C_1 C_2}{C_1 + C_2}$ $L_T = L_1 + L_2 = 2$ Hery

$= \frac{1}{2}$ Farad



$V_c(10\pi^+)$ $\quad$ $V_c(10\pi^+)=1\,\text{Volt}$

$L_T=2H$ $\quad$ $I_{L_1}(10\pi^+)=I_L(10\pi^+)=1\,\text{Amper}$

$$\frac{d^2 I_L}{dt^2}+\frac{1}{RC_T}\frac{dI_L}{dt}+\frac{1}{L_TC_T}I_L=\frac{1}{L_TC_T}I_S(t)$$

$$\frac{d^2 I_L}{dt^2}+I_L=I_S(t)$$

$$I_L(10\pi^+)=1A$$

$$\frac{d^2 V_c}{dt^2}+\frac{1}{RC_T}\frac{dV_c}{dt}+\frac{1}{L_TC_T}V_L=\frac{1}{C_T}\frac{d}{dt}I_S(t)$$

$$V_c(10\pi^+)=1\,\text{Volt}$$

$$\frac{d^2V_c}{dt^2}+V_c=2\frac{d}{dt}e^{-t}=-2e^{-t}$$

$$\frac{dI_L}{dt}(10\pi^+)=\frac{V_c(10\pi^+)}{L_T}=\frac{1}{2}$$

$s^2+1=0$ $\quad$ $s_{1,2}=\mp j$ → natural frequencies → $s_1\neq -1$, $s_2\neq -1$ (dynamic harcheyed)

1. $I_{Lp}=I_{L_1p}=Me^{-t}$

(particular solution is similar to input when $t>10\pi$)

$$\frac{d^2}{dt^2}I_L+I_L=e^{-t}$$

$$\frac{d^2}{dt^2}(Me^{-t})+Me^{-t}=e^{-t}$$

$$2Me^{-t}=e^{-t}\quad M=\frac{1}{2}$$

$\Rightarrow$

$$S_{1,2} = \mp j$$

$$I_{Lh} = A_1 \cos(t) + A_2 \sin(t)$$

$$I_L = I_{cp} + I_{ch}$$

$$I_L = \frac{1}{2} e^{-t} + A_1 \cos(t) + A_2 \sin(t)$$

$$I_L(10\pi^+) = 1 \text{ Amper} = \frac{1}{2} e^{-10\pi} + A_1 \cos(10\pi) + A_2 \sin(10\pi)$$

$$1 = 0 + A_1 + A_2 \Rightarrow \quad A_1 = 1$$

$$\left.\frac{dI_{L1}}{dt}\right|_{t=10\pi^+} = \frac{1}{2}$$

$$\frac{dI_L}{dt} = -\frac{1}{2} e^{-t} - A_1 \sin(t) + A_2 \cos(t)$$

$$\left.\frac{dI_L}{dt}\right|_{t=10\pi^+} = -\frac{1}{2} e^{-10\pi^+} - \sin(10\pi + A_2 \cos(10\pi))$$

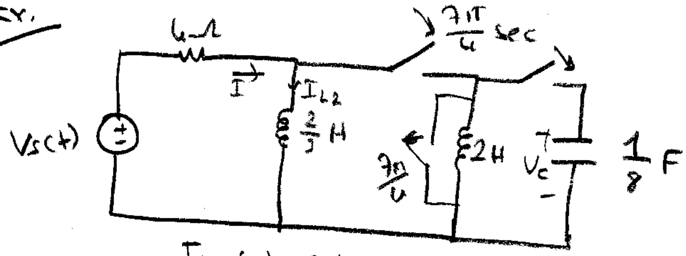
$$\frac{1}{2} = 0 - 0 + A_2$$

$$A_2 = \frac{1}{2}$$

$$I_L(t) = \frac{1}{2} e^{-t} + \cos(t) + \frac{1}{2} \sin(t)$$

$$I_{L1} = \begin{cases} \frac{1}{2} t^2 e^{-t} & 0 < t < 10\pi \\ \frac{1}{2} e^{-t} + \cos(t) + \sin(t) & t > 10\pi \end{cases}$$

Ex.

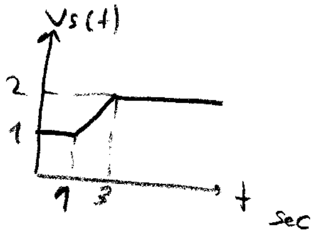


$I_{L1}(0) = 0A$

$I_{L2}(0) = 0A$

$V_C(0) = 3V$

$e^{-x} \cong 0 \quad x \geq 5$

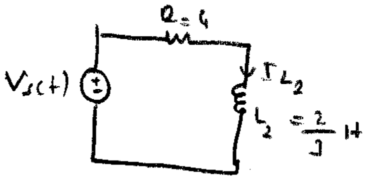


Soltn.

$V_C\left(\frac{7\pi^-}{4}\right) = V_C(0)$ ✓

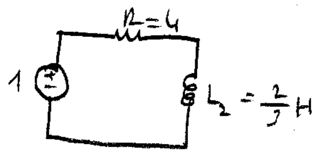
$I_{L1}\left(\frac{7\pi^-}{4}\right) = I_{L1}(0)$ ✓

$0 < t < \frac{7\pi}{4}$



$I_{L2}(0) = 0A$ //

when $0 < t < 1$ $\quad V_s(t) = 1$



$I_{L2} = 0A$

$V_s = V_R + V_{L2}$

$V_s = R I_{L2} + L_2 \frac{dI_{L2}}{dt}$

$1 = 4 I_{L2} + \frac{2}{3} \frac{dI_L}{dt}$

=>

$$\frac{dI_{L_2}}{dt} + 6 I_{L_2} = \frac{3}{2}$$

① $I_{L_{2p}} = k$

$$\frac{d}{dt} k + 6k = \frac{3}{2} \qquad k = \frac{1}{4}$$

② $I_{L_{2h}} = M e^{-6t}$ $\left[\begin{array}{l} s+6=0 \text{ char eq} \\ s=-6 \text{ nat-fre} \end{array} \right]$

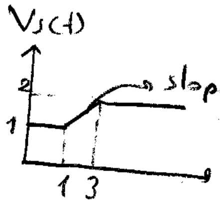
③ $I_{L_2} = I_{L_{2p}} + I_{L_{2h}}$

$$I_{L_2} = \frac{1}{4} + M e^{-6t}$$

$$I_{L_2}(0) = 0 = \frac{1}{4} + M e^{-0} \Rightarrow M = -\frac{1}{4}$$

$I_{L_2} = \frac{1}{4} - \frac{1}{4} e^{-6t}$ when $t=1$ $I_{L_2}(1) = \frac{1}{4} - \frac{1}{4} e^{-6} \approx \frac{1}{4}$

when $1 < t < 3$

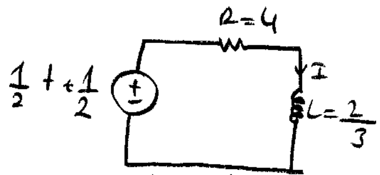


slope $= \frac{2-1}{3-1} = \frac{1}{2}$

$$\frac{1}{2} = \frac{V_s(t) - 2}{t-3}$$

$$\frac{1}{2}(t-3) + 2 = V_s(t)$$

$$\frac{1}{2}t + \frac{1}{2} = V_s(t)$$



$$\frac{1}{2}t + \frac{1}{2} = V_R + V_L$$

$$\frac{1}{2}t + \frac{1}{2} = 4 I_{L_2} + \frac{2}{3} \frac{dI_{L_2}}{dt}$$

$$\frac{3t}{4} + \frac{3}{4} = 6 I_{L_2} + \frac{dI_{L_2}}{dt}$$

$$I_{L_2}(1) = \frac{1}{4}$$

$I_{L_2 p}(t) = At + B$

$$\frac{d}{dt}(At+B) + 6(At+B) = \frac{3t}{4} + \frac{3}{4}$$

$$A + 6At + 6B = \frac{3t}{4} + \frac{3}{4}$$

$$6A = \frac{3}{4} \qquad A + 6B = \frac{3}{4}$$

$$A = \frac{1}{8} \qquad \frac{1}{8} + 6B = \frac{3}{4} \rightarrow 6B = \frac{5}{8} \qquad B = \frac{5}{48}$$

$$I_{L_2 p} = \frac{1}{8}t + \frac{5}{48}$$

$$I_{L_2 h}(t) = \mu_1 e^{-t}$$

$$= \mu_1 e^{-(t-1)}$$

$$I_{L_2} = I_{L_2 h} + I_{L_2 p}$$

$$= \mu_1 e^{-6(t-1)} + \frac{1}{8}t + \frac{5}{48}$$

$$I_{L_2}(1) = \mu_1 + \frac{1}{8} + \frac{5}{48}$$

$$\frac{1}{4} = \mu_1 + \frac{11}{48}$$

$$\mu_1 = \frac{1}{48}$$

$1 < t < 3$

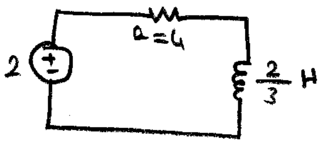
$$I_{L_2}(t) = \frac{1}{48} e^{-6(t-1)} + \frac{1}{8} + \frac{5}{48}$$

$$I_{L_2}(3) = \underbrace{\frac{1}{48} e^{-6(3-1)}}_{0} + \frac{2}{8} + \frac{5}{48}$$

$$I_{L_2}(3) \cong \frac{23}{48}$$

$\Longrightarrow$

$3 < t < \frac{7\pi}{4} \longrightarrow v_s(t) = 2$



$2 = V_R + V_{L_2}$

$2 = I_{L_2} 4 + \frac{2}{3} \frac{dI_{L2}}{dt}$

$3 = \frac{dI_{L2}}{dt} + 6 I_{L_2} \qquad I_{L_2}(3) = \frac{23}{48}$

① $I_{L_2 p} = K$

$\frac{d}{dt} K + 6K = 3$

$K = 1/2$

② $I_{L_2 h} = N_1 e^{-6(t-3)}$

③ $I_{L_2} = I_{L_2 p} + I_{L_2 h}$

$= \frac{1}{2} + N_1 e^{-6(t-3)}$

$I_{L_2}(3) = \frac{1}{2} + N_1 = \frac{23}{48} \Rightarrow N_1 = \frac{-1}{48}$

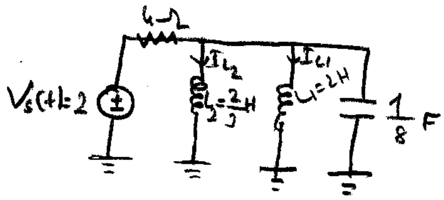
$I_{L_2} = \frac{1}{2} - \frac{1}{48} e^{-6(t-3)}$

$I_{L_2}\left(\frac{7\pi^-}{4}\right) = \frac{1}{2} - \underbrace{\frac{1}{48} e^{-6\left(\frac{7\pi}{4}-3\right)}}_{0}$

$\cong \frac{1}{2}$

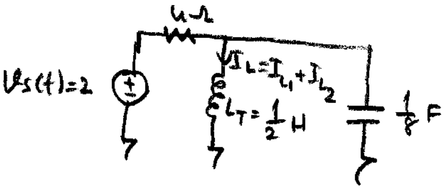
$\Longrightarrow$

f) $\frac{7\pi}{4}$



$V_c\left(\frac{7\pi}{4}^+\right) = V_c\left(\frac{7\pi}{4}^-\right) = V_c(0) = 3$

$L_T = \frac{L_1 L_2}{L_1+L_2} = \frac{2\times\frac{2}{3}}{2+\frac{2}{3}} = \frac{1}{2}\ H$



$I_L\left(\frac{7\pi}{4}^+\right) = \underbrace{I_{L_1}\left(\frac{7\pi}{4}^-\right)}_{0} + \underbrace{I_{L_2}\left(\frac{7\pi}{4}^-\right)}_{\frac{1}{2}} = \frac{1}{2}\ A$

$V_c\left(\frac{7\pi}{4}^+\right) = 3V$

$V_s = V_R + V_c$

$2 = (I_L + I_c)R + V_c$

$2 = (I_L + \frac{1}{8}\frac{dV_c}{dt})R + V_c$

$\boxed{\frac{dV_c}{dt} = 4 - 8I_L - 2V_c}$ $\qquad V_c\left(\frac{7\pi}{4}^+\right) = 3V$

$\frac{1}{2}\frac{dI_L}{dt} = V_L$ $\boxed{\frac{dI_L}{dt} = 2V_c}$ $\qquad I_L\left(\frac{7\pi}{4}^+\right) = \frac{1}{2}A$

$\frac{d^2V_c}{dt^2} = 0 - 8\overbrace{\frac{dI_L}{dt}}^{2V_c} - 2\frac{dV_c}{dt}$

$\boxed{\frac{d^2V_c}{dt^2} + 2\frac{dV_c}{dt} + 16V_c = 0}$

$\Longrightarrow$

$$\frac{d^2 I_L}{dt} = 2\,\frac{dV_c}{dt}$$

$$\frac{d^2 I_L}{dt^2} = 2\left[4 - 8I_L - 2V_c\right]$$

$$\frac{d^2 I_L}{dt^2} + 16 I_L = 8 - 4V_c$$

$$= 8 - \frac{2\,dI_L}{dt}$$

$$\boxed{\frac{d^2 I_L}{dt^2} + 2\,\frac{dI_L}{dt} + 16 I_L = 8}$$

$$I_L\left(\tfrac{3\pi}{4}^+\right) = \tfrac{1}{2}A \qquad V_c\left(\tfrac{3\pi}{4}^+\right) = 3V$$

Find $I_{Lp} = X$

$$\frac{d^2}{dt^2}X + 2\,\frac{d}{dt}X + 16X = 8$$

$$16x = 8 \qquad x = \frac{1}{2}$$

$$s^2 + 2s + 16 = 0 \longrightarrow s_{1,2} = \frac{-2 \mp \sqrt{4-64}}{2} = \frac{-1 \mp \sqrt{-60}}{20}$$

$$s_{1,2} = -1 \pm j\sqrt{15}$$

$$I_L(t) = I_{Lp} + I_{Lh}$$

$$= \frac{1}{2} + e^{-(t-\frac{3\pi}{4})}\left[A_1\cos\left[\sqrt{15}\left(t-\tfrac{3\pi}{4}\right)\right] + A_2\sin\left[\sqrt{15}\left(t-\tfrac{3\pi}{4}\right)\right]\right]$$

$$I_L\left(\tfrac{3\pi}{4}^+\right) = \frac{1}{2} = \frac{1}{2} + 1\left[A_1\underbrace{\cos 0}_{1} + A_2\underbrace{\sin 0}_{0}\right]$$

$$A_1 = 0$$

$$I_L(t) = \frac{1}{2} + e^{-(t-\frac{3\pi}{4})}A_2\sin\left(\sqrt{15}\left(t-\tfrac{3\pi}{4}\right)\right)$$

$$L_T\,\frac{dI_L}{dt} = V_c$$

$$\frac{dI_L}{dt} = \frac{V_c}{L_T}$$

$$\left.\frac{dI_L}{dt}\right|_{t=\frac{3\pi}{4}^+} = \frac{V_c\left(\frac{3\pi}{4}^+\right)}{1/2} = \frac{3}{\frac{1}{2}} = 6$$

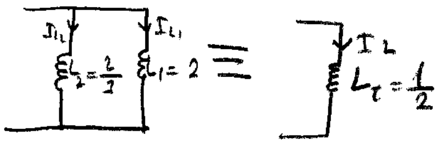
$$\Longrightarrow$$

$$\frac{dI_L}{dt} = 0 - e^{-(t-\frac{7\pi}{4})} \cdot A_2 \sin\left(\sqrt{15}\left(t-\frac{7\pi}{4}\right)\right) + e^{-(t-\frac{7\pi}{4})} A_2 \sqrt{15}\cos\left(\sqrt{15}\left(t-\frac{7\pi}{4}\right)\right)$$

$$\left.\frac{dI_L}{dt}\right|_{t=\frac{7\pi}{4}} = 0 + A_2\sqrt{15} = 6$$

$$A_2 = \frac{6}{\sqrt{15}}$$

$$I_L(t) = \frac{1}{2} + \frac{6}{\sqrt{15}} e^{-(t-\frac{7\pi}{4})} \sin\left(\sqrt{15}\left(t-\frac{7\pi}{4}\right)\right)$$



$$I_{L_2} = I_L \times \frac{L_1}{L_1+L_2} \qquad I_{L_1} = I_L \times \frac{L_2}{L_1+L_2}$$

$$= I_L \frac{2}{2+\frac{2}{3}} \qquad I_{L_1} = I_L \times \frac{\frac{2}{3}}{2+\frac{2}{3}} = I_L \frac{2}{8}$$

$$= I_L \frac{6}{8}$$

$$I_{L_2} = \frac{3}{4}\left[\frac{1}{2} + \frac{6}{\sqrt{15}} e^{-(t-\frac{7\pi}{4})} \sin\left(\sqrt{15}\left(t-\frac{7\pi}{4}\right)\right)\right]$$

$$I_{L_2} = \begin{cases} \frac{1}{4} - \frac{1}{4} e^{-6t} & 0 < t \le 1 \\ \frac{t}{48} + \frac{5}{48} + \frac{1}{48} e^{-6(t-1)} & 1 < t \le 3 \\ \frac{1}{2} - \frac{1}{48} e^{-6(t-3)} & 3 < t < \frac{7\pi}{4} \\ \frac{3}{4}\left[\frac{1}{2} + \frac{6}{\sqrt{15}} e^{-(t-\frac{7\pi}{4})} \sin\left(\sqrt{15}\left(t-\frac{7\pi}{4}\right)\right)\right] & t > \frac{7\pi}{4} \end{cases}$$