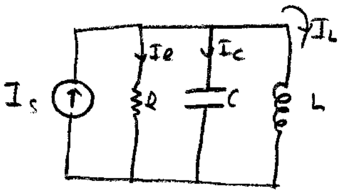
25.07.2010

Step Input ($I_s(t) = u(t)$)



$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + \omega_0^2 V_c = \frac{1}{C}\frac{dI_s}{dt} = \frac{1}{C}\delta(t)$$

$$V_c(0^-) = 0$$

$$\frac{d^2I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + \omega_0^2 I_L = \omega_0^2 I_s = \omega_0^2 u(t)$$

$$I_L(0^-) = 0$$

$$\frac{dV_c}{dt}(0^-) = -\frac{V_c(0^-)}{RC} - \frac{I_L(0^-)}{C} + \frac{\overbrace{I_s(0^-)}^{u(0^-)}}{C}$$

$$= -\frac{0}{RC} - \frac{0}{C} + \frac{0}{C} = 0$$

$$\frac{dI_L}{dt}(0^-) = \frac{V_c(0^-)}{L} = 0$$

$t > 0$

$$\frac{dV_c}{dt} = -\frac{1}{RC}V_c - \frac{1}{C}I_L + \frac{1}{C}I_s$$

$$\int_{V_c(0^-)}^{V_c(0^+)} dV_c = -\frac{1}{RC}\underbrace{\int_{0^-}^{0^+} V_c(t')dt'}_{0} - \frac{1}{C}\underbrace{\int_{0^-}^{0^+} I_L(t')dt'}_{0} + \frac{1}{C}\underbrace{\int_{0^-}^{0^+} \underbrace{I_s(t')}_{u(t)}dt'}_{0}$$

$$V_c(0^+) - V_c(0^-) = 0$$

$$\frac{dI_L}{dt} = \frac{V_c}{L}$$

$$\int_{I_L(0^-)}^{I_L(0^+)} dI_L = \frac{1}{L} \int_{0^-}^{0^+} V_c(t')dt'$$

$$I_L(0^+) - I_L(0^-) = 0 \Rightarrow I_L(0^+) = I_L(0^-) = 0$$

$$\frac{dI_L}{dt}(0^+) = \frac{V_c(0^+)}{L} = \frac{0}{L} = 0$$

$$\frac{dV_c}{dt}(0^+) = -\frac{1}{RC} V_c(0^+) - \frac{1}{C} I_L(0^+) + \frac{1}{C} I_s(0^+)$$

$$= 0 - 0 + \frac{1}{C}$$

$$\boxed{\frac{dV_c}{dt}(0^+) = \frac{1}{C}}$$

$t > 0$

$$\frac{d^2V_c}{dt^2} + 2\alpha \frac{dV_c}{dt} + \omega_0^2 V_c = 0$$

$$V_c(0^+) = 0 \quad \frac{dV_c}{dt}(0^+) = \frac{1}{C}$$

$$\frac{d^2I_L}{dt^2} + 2\alpha \frac{dI_L}{dt} + \omega_0^2 I_L = \omega_0^2$$

$$I_L(0^+) = 0 \quad \frac{dI_L}{dt}(0^+) = 0$$

$$2\alpha = \frac{1}{RC} \qquad \omega_0^2 = \frac{1}{LC}$$

For I_L

Assume overdamped $\rightarrow s_{1,2}$ real, different, negative

① $I_{Lp} = K$ (constant) $I_{cp} \rightarrow$ particular solution

$$\underbrace{\frac{d^2}{dt^2}K}_{0} + 2\alpha \underbrace{\frac{d}{dt}K}_{0} + w_0^2 K = w_0^2$$

$$w_0^2 K = w_0^2 \implies K = 1$$

② $s^2 + 2\alpha s + w_0^2 = 0$ (char eq)

$s = 1,2$ solution

$$I_{LH} = B_1 e^{s_1 t} + B_2 e^{s_2 t}$$

③ $I_L(t) = I_{cp} + I_{cH}$

$$I_L(t) = 1 + B_1 e^{s_1 t} + B_2 e^{s_2 t}$$

④ $I_L(0^+) = 0 = 1 + B_1 + B_2$

$$\frac{dI_L}{dt}(0^+) = 0 = s_1 B_1 + s_2 B_2$$

$$B_1 = \frac{s_2}{s_1 - s_2} \quad B_2 = \frac{s_1}{s_2 - s_1}$$

$$L\frac{dI_L}{dt} = V_C = L\left[\frac{d}{dt}\left[1 + \frac{s_2}{s_1 - s_2}e^{s_1 t} + \frac{s_1}{s_2 - s_1}e^{s_2 t}\right]\right]$$

$$= L\left[\frac{s_1 s_2}{s_1 - s_2}e^{s_1 t} + \frac{s_1 s_2}{s_2 - s_1}e^{s_2 t}\right]$$

$$s_1 s_2 = w_0^2 = \frac{1}{LC}$$

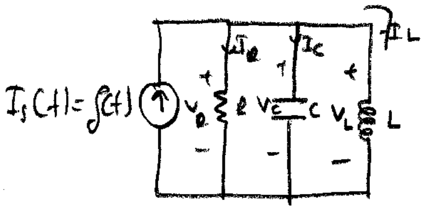
$$= L\left[\frac{\frac{1}{LC}}{s_1 - s_2}e^{s_1 t} + \frac{\frac{1}{LC}}{s_2 - s_1}e^{s_2 t}\right]$$

$$v_C(t) = \left[\frac{1}{L}\frac{1}{(s_1 - s_2)}e^{s_1 t} + \frac{1}{C(s_2 - s_1)}e^{s_2 t}\right]$$

$$\lim_{t\to\infty} V_C(t) = 0 \quad (\text{at steady-state } V_C(t) \to 0)$$

$$\lim_{t\to\infty} I_L(t) = 1 \quad (\text{at steady state } I_L(t) \to 1)$$

<u>Impulse Response</u> $(I_s(t) = \delta(t))$



$$\frac{d^2V_C}{dt^2} + 2\alpha\frac{dV_C}{dt} + \omega_0^2 V_C = \frac{1}{C}\underbrace{\frac{d}{dt}\delta(t)}_{\dot{\delta}(t)}$$

$$V_C(0^-) = 0$$

$$\frac{d^2I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + \omega_0^2 I_L = \omega_0^2 \underbrace{I_S(t)}_{\delta(t)}$$

$$I_L(0^-) = I_0$$

$$\frac{dV_C}{dt}(0^-) = -\frac{1}{RC}V_C(0^-) - \frac{1}{C}I_L(0^-) + \frac{1}{C}I_S(0^-)$$

$$= -\frac{V_0}{RC} - \frac{I_0}{C} + \frac{1}{C}0$$

$$\frac{dV_C}{dt}(0^-) = -\frac{V_0}{RC} - \frac{I_0}{C}$$

$$\frac{dI_L}{dt}(0^-) = \frac{V_C(0^-)}{L} = \frac{V_0}{L}$$

$t>0$

$$\frac{dV_C}{dt} = -\frac{1}{RC}V_C - \frac{1}{C}I_L + \frac{1}{C}I_S$$

$$\int_{V_C(0^-)}^{V_C(0^+)} dV_C = -\frac{1}{RC}\int_{0^-}^{0^+} V_C(t')dt' - \frac{1}{C}\int_{0^-}^{0^+} I_L(t')dt' + \frac{1}{C}\int_{0^-}^{0^+} \underbrace{I_S(t)}_{\delta(t)}\,dt$$

$$V_C(0^+) - V_C(0^-) = 0 \quad - \quad 0 \quad + \frac{1}{C}1 = \frac{1}{C}$$

$$V_C(0^+) = V_C(0^-) + \frac{1}{C}$$

$$V_C(0^+) = V_0 + \frac{1}{C}$$

$$\frac{dI_L}{dt} = \frac{V_c}{L}$$

$$\int_{I_L(0^-)}^{I_L(0^+)} dI_L = \frac{1}{L}\int_{0^-}^{0^+} V_c(t')\,dt'$$

$$I_L(0^+) - I_L(0^-) = 0$$

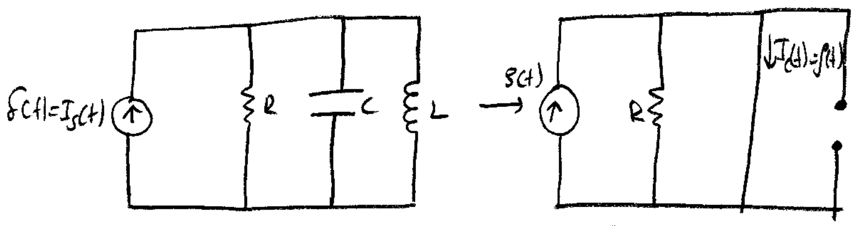
$$I_L(0^+) = I_L(0^-) = I_0 \,_{/\!/}$$

$$\frac{dI_L}{dt}(0^+) = \frac{V_c(0^+)}{L} = \frac{V_0 + \frac{1}{C}}{L} = \frac{V_0}{L} + \frac{1}{LC}$$

$$\frac{dV_c}{dt}(0^+) = -\frac{1}{RC}V_c(0^+) - \frac{1}{C}I_L(0^+) + \frac{1}{C}\underbrace{I_s(0^+)}_{\delta(t)}$$

$$= -\frac{1}{RC}\left(V_0 + \frac{1}{C}\right) - \frac{1}{C}I_0 + 0$$

$$= \frac{-V_0}{RC} - \frac{I_0}{C} - \frac{1}{C^2R}$$



$$I_c(t) = \delta(t)$$

$$C\frac{dV_c}{dt} = I_c(t) = \delta(t)$$

$$\int_{V_c(0^-)}^{V_c(0^+)} dV_c = \frac{1}{C}\int_{0^-}^{0^+} \underbrace{I_c(t')}_{\delta(t)}\,dt'$$

$$V_c(0^+) - V_c(0^-) = \frac{1}{C}\cdot 1$$

$$V_c(0^+) - \underbrace{V_c(0^-)}_{V_0} = \frac{1}{C}$$

$$V_c(0^+) = V_0 + \frac{1}{C}$$

$t>0$

$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + \omega_0^2 V_c = 0$$

$$V_c(0^+) = V_0 + \frac{1}{C} \qquad \frac{dV_c}{dt}(0^+) = -\frac{V_0}{RC} - \frac{I_0}{C} - \frac{1}{RC^2}$$

$$\frac{d^2I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + \omega_0^2 I_L = 0$$

$$I_L(0^+) = I_0 \qquad \frac{dI_L}{dt}(0^+) = \frac{V_0}{L} + \frac{1}{LC}$$

Let $V_0 = 0$, $I_0 = 0$

s_1, s_2 real, negative, different (overdamped)

① $I_{LP} = 0$

② $I_{LH} = B_1 e^{s_1 t} + B_2 e^{s_2 t}$

③ $I_L = I_{LH} + I_{LP} = B_1 e^{s_1 t} + B_2 e^{s_2 t}$

④ $I_L(0^+) = 0 = B_1 + B_2$

$$\frac{dI_L}{dt}(0^+) = \frac{1}{LC} = s_1 B_1 + s_2 B_2 = \omega_0^2$$

$$B_1 = \frac{\omega_0^2}{s_1 - s_2}, \quad B_2 = \frac{\omega_0^2}{s_2 - s_1}$$

$$L\frac{dI_L}{dt} = V_c \implies L\left[\frac{d}{dt}\left[\frac{\omega_0^2}{s_1-s_2}e^{s_1 t} + \frac{\omega_0^2}{s_2-s_1}e^{s_2 t}\right]\right]$$

$$V_c = L\left[\frac{s_1\omega_0^2}{s_1-s_2}e^{s_1 t} + \frac{s_2\omega_0^2}{s_2-s_1}e^{s_2 t}\right]$$

$$= \left[\frac{1}{C}\frac{s_1}{s_1-s_2}e^{s_1 t} + \frac{s_2}{C(s_2-s_1)}e^{s_2 t}\right]$$

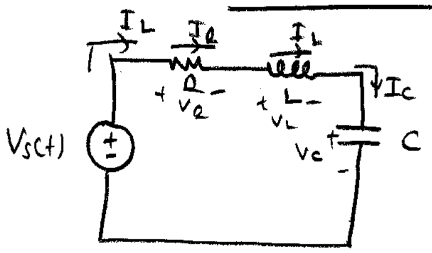
if $V_0 \neq 0$ $I_0 \neq 0$

$$I_L(0^+) = I_0 = B_1 + B_2$$

$$\frac{dI_L}{dt}(0^+) = \frac{V_0}{L} + \frac{1}{LC} = s_1 B_1 + s_2 B_2 \qquad \text{HW!!!}$$

* Note that, the zero-state unit step response is the time derivative of zero-state ramp response AND zero-state impulse response is the time derivative of zero-state unit step response.

Series RLC



$$V_C(t_0) = V_0$$

$$I_L(t_0) = I_0$$

$$I_R = I_L = I_C$$

$$I_C = C\frac{dV_C}{dt} = I_L \Rightarrow \boxed{\frac{dV_C}{dt} = \frac{I_L}{C}} \quad (1)$$

$$V_s(t) = V_R + V_L + V_C$$

$$V_s(t) = I_R R + L\frac{dI_L}{dt} + V_C$$

$$V_s(t) = I_L R + L\frac{dI_L}{dt} + V_C$$

$$\frac{dI_L}{dt} = -\frac{1}{L}V_C - \frac{R}{L}I_L + \frac{V_s}{L}$$

$$R = \frac{1}{G} \Rightarrow \boxed{\frac{dI_L}{dt} = -\frac{1}{L}V_C - \frac{1}{LG}I_L + \frac{V_s}{L}} \quad (2)$$

$G \rightarrow$ conductance

$$\frac{d}{dt}\begin{bmatrix} V_C \\ I_L \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{1}{LG} \end{bmatrix}\begin{bmatrix} V_C \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} V_s$$

$$V_C(t_0) = V_0$$

$$I_L(t_0) = I_0$$

$\Rightarrow$

Take derivative of ① put ② inside;

$$= \frac{d^2V_c}{dt^2} = \frac{1}{C}\frac{dI_L}{dt} = \frac{1}{C}\left[-\frac{1}{L}V_c - \frac{1}{LG}I_L + \frac{1}{L}V_s\right]$$

$$= \frac{d^2V_c}{dt^2} = -\frac{1}{LC}V_c - \frac{1}{LCG}\underbrace{I_L}_{C\frac{dV_c}{dt}} + \frac{1}{LC}V_s$$

$$= \boxed{\frac{d^2V_c}{dt^2} + \frac{1}{LG}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{1}{LC}V_s}$$

$$V_c(t_0) = V_0$$

$$\frac{dV_c}{dt}(t_0) = \frac{I_L(t_0)}{C} = \frac{I_0}{C}$$

Take derivative of ② put ① inside;

$$\frac{d^2I_L}{dt^2} = -\frac{1}{L}\underbrace{\frac{dV_c}{dt}}_{\frac{I_L}{C}} - \frac{1}{LG}\frac{dI_L}{dt} + \frac{1}{L}\frac{dV_s}{dt}$$

$$\frac{d^2I_L}{dt^2} + \frac{1}{LG}\frac{dI_L}{dt} + \frac{1}{LC}I_L = \frac{1}{L}\frac{dV_s}{dt}$$

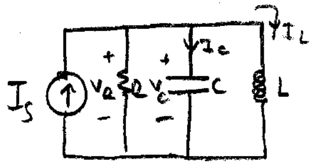
$$I_L(t_0) = I_0 \qquad \frac{dI_L}{dt}(t_0) = -\frac{1}{L}V_c(t_0) - \frac{1}{LG}I_L(t_0) + \frac{1}{L}V_s(t_0)$$

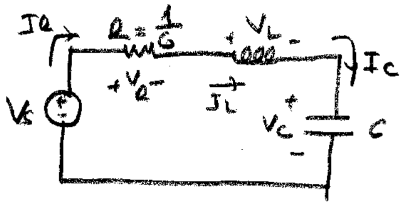
$$= -\frac{V_0}{L} - \frac{I_0}{LG} + \frac{V_s(t_0)}{L}$$

$$\frac{1}{LG} = \frac{R}{L} = 2\alpha \qquad \alpha\text{: damping ratio}$$

$$\frac{1}{LC} = w_0^2 \qquad w_0\text{: rezonans frequency}$$

Parallel RLC $\xleftrightarrow{\text{Dual Circuit}}$ Series RLC

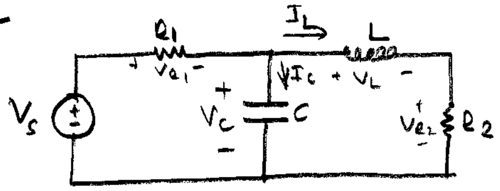




Transformation

parallel	series
I_s	V_s
R	G
L	C
C	L
I_L	V_C
V_C	I_L
V_0	I_0
I_0	V_0

Ex.



$V_C(0^-) = V_0$

$I_L(0^-) = I_0$

Solution.

$V_s = V_{R_1} + V_C$

$V_s = R_1[I_L + I_C] + V_C$

$V_s = R_1 I_L + R_1 C \frac{dV_C}{dt} + V_C$

① $\boxed{\frac{dV_C}{dt} = \frac{V_s}{R_1 C} - \frac{1}{C} I_L - \frac{V_C}{R_1 C}}$

$V_C = V_L + V_{R_2}$

$V_C = L\frac{dI_L}{dt} + R_2 I_L$

② $\boxed{\frac{dI_L}{dt} = \frac{1}{L} V_C - \frac{R_2}{L} I_L}$

$$\frac{d}{dt}\begin{bmatrix} V_C \\ I_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_1 C} & \frac{-1}{C} \\ \frac{1}{L} & \frac{-R_2}{L} \end{bmatrix}\begin{bmatrix} V_C \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C} \\ 0 \end{bmatrix} V_s$$

$\Longrightarrow$

Take derivative of ① put ② inside;

$$\frac{d^2V_c}{dt^2} = -\frac{1}{R_1C}\frac{dV_c}{dt} - \frac{1}{C}\frac{dI_L}{dt} + \frac{1}{R_1C}\frac{dV_s}{dt}$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{R_1C}\frac{dV_c}{dt} = -\frac{1}{C}\left[\frac{1}{L}V_c - \frac{R_2}{L}I_L\right] + \frac{1}{R_1C}\frac{dV_s}{dt}$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{R_1C}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{R_2}{LC}I_L + \frac{1}{R_1C}\frac{dV_s}{dt}$$

$$I_L = \frac{V_s}{R_1} - C\frac{dV_c}{dt} - \frac{V_c}{R_1}$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{R_1C}\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{R_2}{R_1LC}V_s - \frac{R_2}{L}\frac{dV_c}{dt} - \frac{R_2}{R_1LC}V_c + \frac{1}{R_1C}\frac{dV_s}{dt}$$

$$\boxed{\frac{d^2V_c}{dt^2} + \left(\frac{1}{R_1C} + \frac{R_2}{L}\right)\frac{dV_c}{dt} + \left(\frac{1}{LC} + \frac{R_2}{R_1LC}\right)V_c = \frac{R_2}{R_1LC}V_s + \frac{1}{R_1C}\frac{dV_s}{dt}}$$

$$V_c(0^-) = V_0, \quad \frac{dV_c}{dt}(0^-) = \frac{V_s(0^-)}{R_1C} - \frac{1}{C}I_L(0^-) - \frac{V_c(0^-)}{R_1C}$$

Take derivative of ② put ① inside;

$$\frac{d^2I_L}{dt^2} = \frac{1}{L}\frac{dV_c}{dt} - \frac{R_2}{L}\frac{dI_L}{dt}$$

$$\frac{d^2I_L}{dt^2} + \frac{R_2}{L}\frac{dI_L}{dt} = \frac{1}{L}\left[\frac{V_s}{R_1C} - \frac{1}{C}I_L - \frac{V_c}{R_1C}\right]$$

$$\frac{d^2I_L}{dt^2} + \frac{R_2}{L}\frac{dI_L}{dt} + \frac{1}{LC}I_L = \frac{V_s}{LR_1C} - \frac{V_c}{LR_1C} \qquad L\frac{dI_L}{dt} + R_2I_L$$

$$\frac{d^2I_L}{dt^2} + \frac{R_2}{L}\frac{dI_L}{dt} + \frac{1}{LC}I_L = \frac{V_s}{LR_1C} - \frac{1}{LR_1C}\left[L\frac{dI_L}{dt} + R_2I_L\right]$$

$$\boxed{\frac{d^2I_L}{dt^2} + \left(\frac{R_2}{L} + \frac{1}{R_1C}\right)\frac{dI_L}{dt} + \left(\frac{1}{LC} + \frac{R_2}{R_1LC}\right)I_L = \frac{V_s}{LR_1C}}$$

$$I_L(0^-) = I_0 \qquad \frac{dI_L}{dt}(0^-) = \frac{1}{L}V_c(0^-) - \frac{R_2}{L}I_L(0^-) = \frac{V_0}{L} - \frac{R_2I_0}{L}$$

If $V_s(t) = \delta(t)$ find initial condition changes.

$$\frac{dI_L}{dt} = \frac{1}{L} V_c - \frac{R_2}{L} I_L$$

$$\int_{I_L(0^-)}^{I_L(0^+)} dI_L = \underbrace{\frac{1}{L}\int_{0^-}^{0^+} V_c(t')dt'}_{0} - \underbrace{\frac{R_2}{L}\int_{0^-}^{0^+} I_L(t')dt'}_{0}$$

$$I_L(0^+) - I_L(0^-) = 0 - 0$$

$$\boxed{I_L(0^+) = I_L(0^-) = I_0}$$

integrate

$$\frac{dV_c}{dt} = -\frac{1}{R_1C} V_c - \frac{1}{C} I_L + \frac{1}{R_1C} V_s(t)$$

$$\int_{V_c(0^-)}^{V_c(0^+)} dV_c = \underbrace{-\frac{1}{R_1C}\int_{0^-}^{0^+} V_c(t')dt' - \frac{1}{C}\int_{0^-}^{0^+} I_L(t')dt'}_{0} + \underbrace{\frac{1}{R_1C}\int_{0^-}^{0^+} \underbrace{V_s(t')}_{\delta(t)}dt'}_{\frac{1}{R_1C}}$$

$$V_c(0^+) - V_c(0^-) = 0 + \frac{1}{R_1C}$$

$$V_c(0^+) = V_c(0^-) + \frac{1}{R_1C} = V_0 + \frac{1}{R_1C}$$

$$\frac{dI_L}{dt}(0^+) = \frac{1}{L} V_c(0^+) - \frac{R_2}{L} I_L(0^+) = \frac{1}{L}\left(V_0 + \frac{1}{R_1C}\right) - \frac{R_2}{L} I_0 = \frac{V_0}{L} + \frac{1}{LR_1C} - \frac{R_2 I_0}{L}$$

$$\frac{dV_c}{dt}(0^+) = -\frac{1}{R_1C} V_c(0^+) - \frac{1}{C} I_L(0^+) + \frac{1}{R_1C} V_s(0^+)$$

$$= -\frac{1}{R_1C}\left[V_0 + \frac{1}{RC}\right] - \frac{1}{C} I_0 + 0$$

$$= -\frac{V_0}{R_1C} - \frac{I_0}{C} - \frac{1}{R^2C^2}$$

$\Rightarrow$

$$\frac{dV_c}{dt} = -\frac{1}{R_1C}V_c - \frac{1}{C}I_L + \frac{1}{R_1C}V_s(t)$$

$$2\alpha = \left(\frac{1}{R_1C} + \frac{R_2}{L}\right)$$

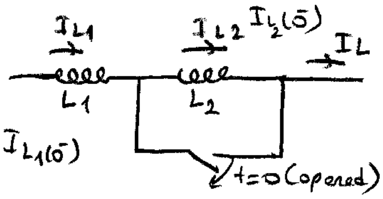
$$\omega_0^2 = \frac{1}{LC} + \frac{R_2}{R_1LC}$$

$t>0$

$$\frac{d^2V_c}{dt^2} + 2\alpha\frac{dV_c}{dt} + \omega_0^2 V_c = 0$$

$$\frac{d^2I_L}{dt^2} + 2\alpha\frac{dI_L}{dt} + \omega_0^2 I_L = 0$$

Switching

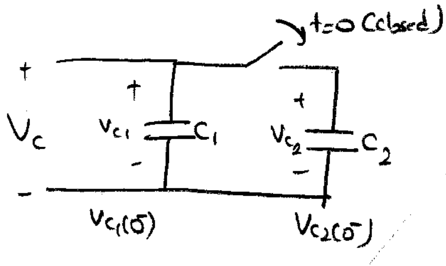


$\phi(0^-) = \phi(0^+) \rightarrow$ flux is preserved

$$I_{L_1}(0^-)L_1 + I_{L_2}(0^-)L_2 = (L_1+L_2)I_L(0^+)$$

$$I_L(0^+) = \frac{I_{L_1}(0^-)L_1 + I_{L_2}(0^-)L_2}{L_1+L_2}$$

$E(0^-) > E(0^+)$ Hw.



$Q(0^-) = Q(0^+)$

choge is preserved

$$\rightarrow V_{c1}(0^-)C_1 + V_{c_2}(0^-)C_2 = (C_1+C_2)V_c(0^+)$$

$$V_c(0^+) = \frac{V_{c1}(0^-)C_1 + V_{c2}(0^-)C_2}{C_1+C_2}$$

$$E(0^-) > E(0^+)$$