#ECE 233#

29.06.2010

Basic Definitions:

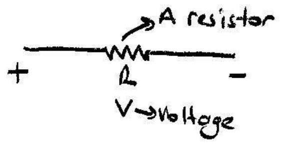
Voltage: The potential difference btw. any two points in an electrical circuit.

Current: The amount of charge flow btw. any two connected points in a circuit.

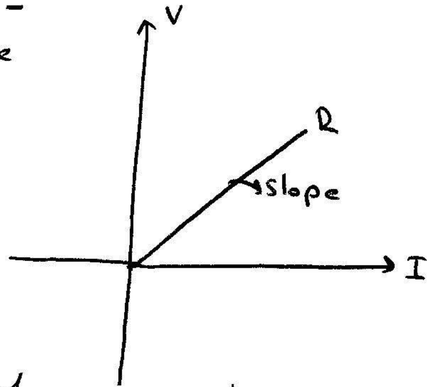
Resistance: The amount of opposition to current flow.

Resistor: A circuit element causing resistance.

Ohm's Law: The relation btw. the voltage drop resistor and current flow over the same resistor is a linear relation.



$\frac{V}{I} = R$ (Ohm's Law)



Conductance: $\frac{1}{\text{Resistance}}$ (siemens)

R → Ohm

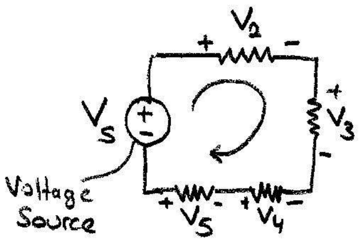
I → Amper

V → Volt

Simple Circuit Theorems

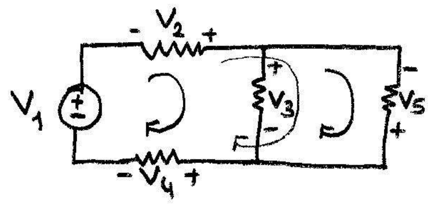
Kirchoff's Voltage Law (KVL);

The algebric sum of all the voltage drops over closed planer circuit is 0.



Applying KVL;

$-V_S + V_2 + V_3 - V_4 - V_5 = 0$



Applying KVL;

$-V_1 - V_2 + V_3 + V_4 = 0$
$V_1 + V_2 - V_3 - V_4 = 0$ } Same!

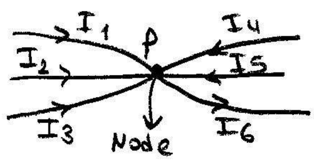
$-V_3 - V_5 = 0 \Rightarrow V_3 = -V_5$

$-V_1 - V_2 - V_5 + V_4 = 0$

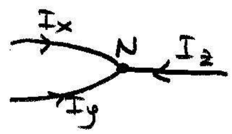
Kirchoff's Current Law;

The sum of incoming currents to a point in a circuit is equal to the sum of outgoing currents from the same point.

Point (Node); Points where two or more circuit elements are connected to each other.

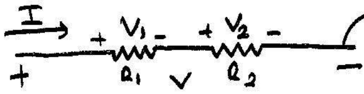


$\underbrace{I_1 + I_2 + I_3 + I_4 + I_5}_{\text{incoming}} = \underbrace{I_6}_{\text{outgoing}}$



$I_x + I_y + I_z = 0$

Series Connection of Resistors



→ Branch : A way that current flows.

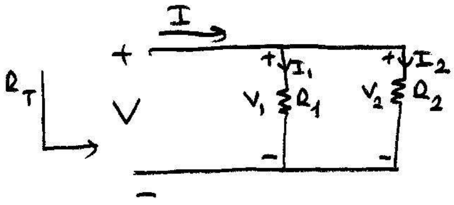
∗ From the same branch, same amounts of current passes.

$$V = V_1 + V_2$$

$$R_T I = R_1 I + R_2 I$$

$$R_T = R_1 + R_2$$

Parallel Connection of Resistors

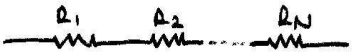


$V_1 = V_2 = V$ (For Parallel Branches)

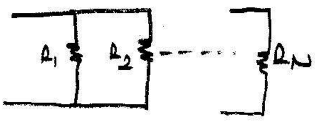
$$I = I_1 + I_2$$

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{OR} \quad R_T = \frac{R_1 R_2}{R_1 + R_2}$$

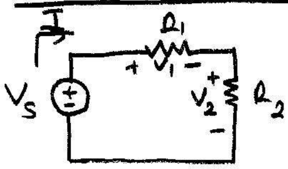


$$R_T = R_1 + R_2 - \cdots R_N$$



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

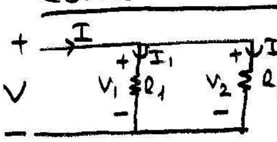
Voltage Division in Series Circuit



$$I = \frac{V_s}{R_1 + R_2} \qquad V_1 = \frac{R_1}{R_1 + R_2} V_s$$

$$V_1 = I \cdot R_1 \qquad V_2 = \frac{R_2}{R_1 + R_2} V_s$$

Current Division in Parallel Circuit



$$V_1 = V_2 = V$$

$$V_1 = I_1 R_1 = V_2 = I_2 R_2$$

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = V_1$$

$$I R_T = I_1 R_1$$

$$I \frac{R_1 R_2}{R_1 + R_2} = I_1 R_1$$

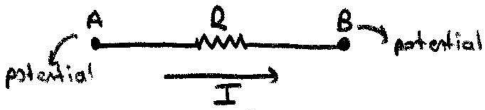
$$\boxed{I_1 = \frac{R_2}{R_1 + R_2} I}$$

$$I_2 R_2 = I R_T$$

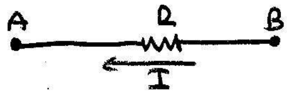
$$I_2 R_2 = I \frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{I_2 = \frac{R_1}{R_1 + R_2} I}$$

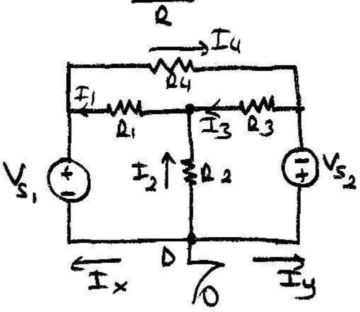
Node Voltage Method



$$I = \frac{A-B}{R}$$



$$I = \frac{B-A}{R}$$



$A - D = V_{s1}$ $\quad C - D = -V_{s2}$

$A - 0 = V_{s1}$ $\quad C = -V_{s2}$

$A = V_{s1}$

① Assign voltage values to all the nodes

② Choose a node as the reference node
(Generally the nodes, which are close to the negative polarity of the sources are chosen as the reference node)

Reference Node = 0 Potential

③ If it's possible using the reference node determine potentials of other nodes.

A ✓ C ✓ D ✓

④ Assign currents to branches of the circuit.

⑤ Use KCL for the nodes where the potential is unknown.

$$I_2 + I_3 = I_1$$

$$\frac{D-B}{R_2} + \frac{C-B}{R_3} = \frac{B-A}{R_1}$$

$D = 0$
$A = V_{s1}$
$C = -V_{s2}$

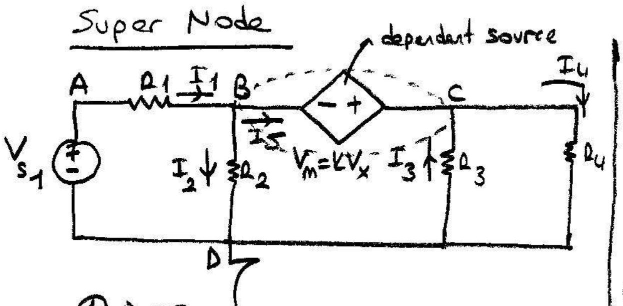
$$\boxed{\frac{-B}{R_2} + \frac{-V_{s2}-B}{R_3} = \frac{B-V_{s1}}{R_1}}$$

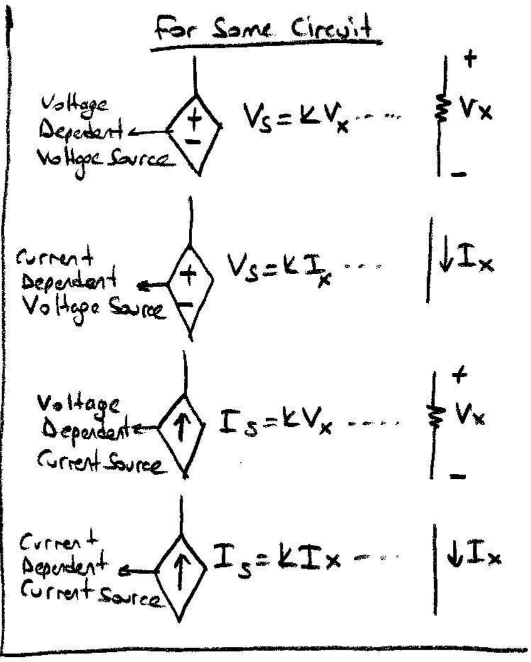
$B \rightarrow$ unknown (solve for B)

$$I_x = I_2 - I_1 = \frac{A-C}{R_1} - \frac{B-A}{R_1}$$

$$I_y = I_3 - I_4 = \frac{C-B}{R_3} - \frac{A-C}{R_4}$$

Super Node





① $D = 0$

② $A = V_{s_1}$

$V_m = kV_x$

③ $C - B = k(C - D)$

$I_S = I_1 - I_2 = I_4 - I_3$ (Super Node eq.)

$I_1 + I_3 = I_2 + I_4$ (Same eq.)

If some voltage source exists btw. two nodes of a circuit that branch acts as super-node and node-voltage eq. is written accordingly.

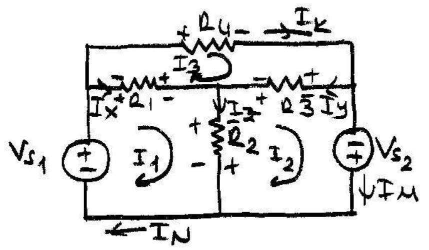
$$I_1 + I_3 = I_2 + I_4$$

④ $$\frac{A-B}{R_1} + \frac{D-C}{R_3} = \frac{B-D}{R_2} + \frac{C-D}{R_4}$$

Mesh Current Method

Mesh: Directed current loop (clockwise direction)

loop current ≠ branch current



① Assign mesh currents (loop currents) for all seperate mesh.

② Determine polarity of circuit elements (except for the sources since they are directly given) according to mesh-current directions.

③ Use KVL to write voltage drops for each seperate mesh.

Mesh I → $-V_{s_1} + R_1(I_1 - I_3) + R_2(I_1 - I_2) = 0$

Mesh II → $-V_{s_2} + R_2(I_2 - I_1) + R_3(I_2 - I_3) = 0$

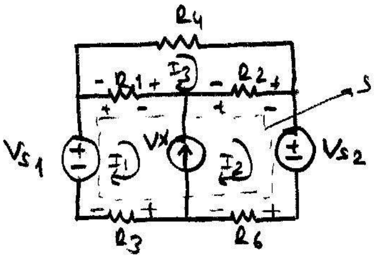
Mesh III → $R_4 I_3 + R_3(I_3 - I_2) + R_1(I_3 - I_1) = 0$

Solve for I_1, I_2, I_3

$I_M = I_2$	$I_x = I_1 - I_3$
$I_N = I_1$	$I_y = I_3 - I_2$
$I_K = I_3$	$I_Z = I_1 - I_2$

branch currents

Super Mesh



Mesh I $\longrightarrow$ $-V_{s_1} + R_1(I_1 - I_3) + V_x + R_3 I_1 = 0$

Mesh II $\longrightarrow$ $V_{s_2} + R_6(I_2) - V_x + R_2(I_2 - I_3) = 0$

Mesh III $\longrightarrow$ $R_4 I_3 + R_2(I_3 - I_2) + R_1(I_3 - I_1) = 0$

Let's eliminate V_x using Mesh I and Mesh II;

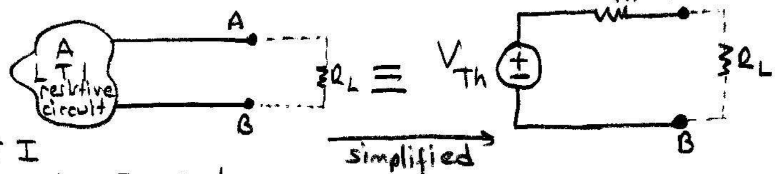
Super Mesh Eq.

$$-V_{s_1} + R_1(I_1 - I_3) + R_2(I_2 - I_3) + V_{s_2} + R_6 I_2 + R_3 I_1 = 0$$

* If there exists a current source btw. two meshes, these two mesh behave a single mesh and a supermesh eq. is written as;

$$\boxed{I_s = I_2 - I_1} \quad **$$

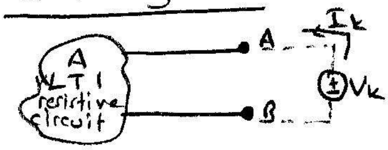
Thevenin Equivalent Circuit



L T I
↳ Linear Time-Invarient

V_{Th} = Open circuit voltage btw. A and B points
R_{Th} = Resistence btw A and B points.

For Finding R_{Th}

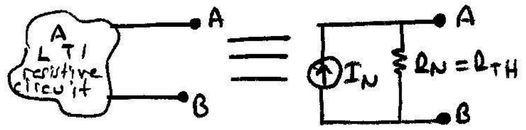


① Kill all independent voltage-current sources.
* Killing voltage source $\longrightarrow$ Short that branch
* " " " $\longrightarrow$ Open " "
Put a source btw. A and B
Calculate the current "I_k"

$$R_{Th} = \frac{V_k}{I_k}$$

② If no dependent source exist in the circuit, kill independent sources and then use series-parallel connection concepts of the resistors to find R_{Th}.

Norton Equivalent Circuit

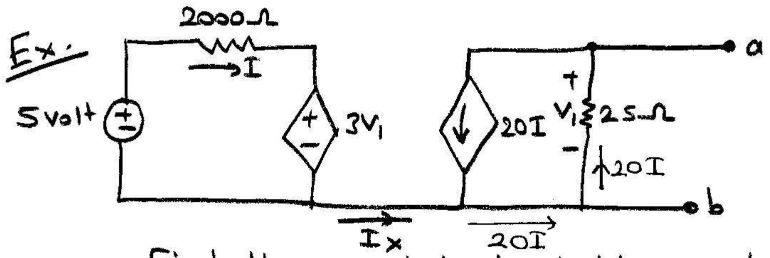


I_N = Short circuit current btw. A-B

$R_N = R_{Th}$ < $R_{Thevenin}$, R_{Norton}

(3) $R_N = R_{Th} = \dfrac{V_{Th}}{I_N}$

Ex.



Find the equivalent circuit btw. a and b.

Solution

For $V_{th} = V_{ab}$ (When a-b is open)

$I = I + I_x$

$I_x = 0$ ✓

$5 = I \cdot 2000 + 3V_1$

$V_1 = -25 \times 20I$

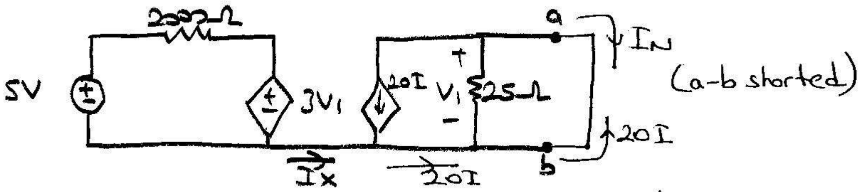
$\boxed{V_1 = -500I}$

$V_{Th} = V_{ab} = V_1$

$I = \dfrac{1}{100}$ Amp

$V_1 = -5$ Volt

$V_{Th} = V_1 = -5$ Volt



To Find I_N

$I = I + I_x$

$I_x = 0$

$V_1 = V_a - V_b = V_{ab} = 0$

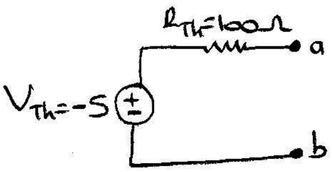
$V_1 = 0 \rightarrow 3V_1 = 0$

$I = \dfrac{5 - 3V_1}{2000} = \dfrac{1}{400}$ Amper

$I_x = \dfrac{V_1}{25} = \dfrac{0}{25} = 0$

$I_N = -20I = -20 \cdot \left(\dfrac{1}{400}\right) = -\dfrac{1}{20}$ A

$R_{Th} = R_N = \dfrac{V_{th}}{I_N} = \dfrac{-5}{-1/20} = 100\,\Omega$



Thevenin equivalent circuit